Closure conversion
or: values representation for functions

Advanced Compiler Construction
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Higher-order functions
A **higher-order function** (HOF) is a function that either:
- takes another function as argument, or
- returns a function.

Many languages offer higher-order functions, but not all provide the same power...
In C, it is possible to pass a function as an argument, and to return a function as a result. However, C functions cannot be nested: they must all appear at the top level. This severely restricts their usefulness, but greatly simplifies their implementation — they can be represented as simple code pointers.
In functional languages – Scala, OCaml, Haskell, etc. – functions can be nested, and they can survive the scope that defined them. This is very powerful as it permits the definition of functions that return “new” functions – e.g. functional composition. However, as we will see, it also complicates the representation of functions, as simple code pointers are no longer sufficient.
To illustrate the issues related to the representation of functions in a functional language, we will use the following L3 example:

```
(def make-adder
  (fun (x)
    (fun (y) (+ x y))))
(def increment (make-adder 1))
(increment 41) ⇒ 42

(def decrement (make-adder -1))
(decrement 42) ⇒ 41
```
Representing adders

To represent the functions returned by make-adder, there are basically two choices:

1. Use simple code pointers. Unfortunately, this implies run-time code generation, as each function returned by make-adder is different!
2. Find another representation for functions, which does not depend on run-time code generation.
Closures
Closures

To adequately represent the functions returned by `make-adder`, their code pointer must be augmented with the value of `x`. Such a combination of a code pointer and an environment giving the values of the free variable(s) – here `x` – is called a closure. The name refers to the fact that the pair composed of the code pointer and the environment is closed, i.e. self-contained.
The code of the closure must be evaluated in its environment, so that \(x\) is “known”.

\[
\text{(make-adder 1)}
\]

\[
\text{(make-adder -1)}
\]

\[
\text{code pointer}
\]

\[
\text{environment}
\]

\[
x \to 1
\]

\[
\text{compiled code for}
\]

\[
\text{(fun (y) (@+ x y))}
\]

\[
\text{code pointer}
\]

\[
\text{environment}
\]

\[
x \to -1
\]

\[
\text{shared code}
\]
Introducing closures

Using closures instead of function pointers changes the way functions are manipulated at run time:
- function abstraction builds and returns a closure instead of a simple code pointer,
- function application extracts the code pointer from the closure, and invokes it with the environment as an additional argument.
Representing closures

During function application, nothing is known about the closure being called – it can be any closure in the program. The code pointer must therefore be at a known and constant location so that it can be extracted. The values contained in the environment, however, are not used during application itself: they will only be accessed by the function body. This provides some freedom to place them.
Flat closures

In flat (or one-block) closures, the environment is “inlined” into the closure itself, instead of being referred from it. The closure itself plays the role of the environment.

```
(make-adder 1)
```

![Diagram of flat closure](image)
Exercise

Given the following L₃ composition function:

```
(def compose
  (fun (f g)
    (fun (x) (f (g x))))))
```

draw the flat closure returned by the application

```
(compose succ twice)
```

assuming that succ and twice are two functions defined in an enclosing scope.
Compiling closures
In a compiler, closures can be implemented by a simplification phase, called closure conversion. Closure conversion transforms a program in which functions can have free variables into an equivalent one containing only closed functions. The output of closure conversion is therefore a program in which functions can be represented as code pointers.
Closure conversion is nothing more than values representation for functions: it encodes the high-level notion of functions of the source language using the low-level concepts of the target language – in this case heap-allocated blocks and code pointers.
The **free variables** of a function are the variables that are used but not defined in that function – i.e. they are defined in some enclosing scope.

The `make-adder` example contains two functions:

```lisp
(def make-adder
  (fun (x)
       (fun (y) (@+ x y))))
```

The outer one does not have any free variable: it is a closed function. The inner one has a single free variable: `x`. 

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Closing functions

Functions are closed by adding a parameter representing the environment, and using it in the function’s body to access free variables.

Function abstraction and application must of course be adapted accordingly:

- abstraction must create and initialize the closure,
- application must pass the environment as an additional parameter.
Assuming the existence of abstract closure-make and closure-get functions, a closure conversion phase could transform the make-adder example as follows:

\[
\text{(def make-adder (fun (x) (fun (y) (@+ x y))))}
\]
\[
\text{(make-adder 1)}
\]

\[
\text{(def make-adder (closure-make (fun (env1 x) (closure-make (fun (env2 y) (@+ (closure-get env2 1) y)) x))))
\]
\[
\text{((closure-get make-adder 0) make-adder 1)}
\]
Recursive closures

Recursive functions need access to their own closure. For example:

```plaintext
(letrec ((f (fun (l) ... (map f l) ...)))
  ...)
```

Several techniques can be used to give a closure access to itself:

- the closure – here $f$ – can be treated as a free variable, and put in its own environment – leading to a cyclic closure,
- the closure can be rebuilt from scratch,
- with flat closures, the environment is the closure, and can be reused directly.
Mutually-recursive closures

Mutually-recursive functions all need access to the closures of all the functions in the definition. For example, in the following program, \( f \) needs access to the closure of \( g \), and the other way around:

\[
\begin{align*}
\text{letrec} & \quad ((f \ (\text{fun} \ (l) \ldots \ (\text{compose} \ f \ g) \ldots)) \\
& \quad \quad \quad \quad\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad\]
Mutually-recursive closures

cyclic closures

- closure for f
  - code ptr. f
    - v_1
    - v_2
    - v_3
  - closure for g
    - code ptr. g
      - w_1
      - w_2

shared closures

- closure for f
  - code ptr. f
- closure for g
  - code ptr. g
    - v_1
    - v_2
    - v_3
    - w_1
    - w_2
CPS/L\textsubscript{3}
closure conversion
In the L₃ compiler, we represent L₃ functions using flat closures. Flat closures are simply blocks tagged with a tag reserved for functions – we choose 202. The first element of the block contains the code pointer while the other elements – if any – contain the environment of the closure.
CPS/L₃ closure conversion

In the L₃ compiler, closure conversion is not a separate phase. Rather, it is the part of the values representation phase that takes care of representing function values. Closure conversion is therefore specified exactly like the values representation phase.
CPS/L₃ free variables

The F function computes the free variables of a CPS/L₃ term:

\[
F[(let_1 ((n \ l)) e)] = F[e] \setminus \{ n \}
\]

\[
F[(let_p ((n (p n_1 ...))) e)] =
\quad (F[e] \setminus \{ n \}) \cup \{ n_1, ... \}
\]

\[
F[(let_c ((n (cnt (a_1 ...) b))) e)] =
\quad F[e] \cup (F[b] \setminus \{ a_1, ... \})
\]

\[
F[(let_f ((f_1 (fun (c_1 n_{1,1} ...) e_1)) ...) e)] =
\quad (F[e] \cup (F[e_1] \setminus \{ n_{1,1}, ... \}) \cup ...) \setminus \{ f_1, ... \}
\]

\[
F[(app_c c n_1 ...)] = \{ n_1, ... \}
\]

\[
F[(app_f f c n_1 ...)] = \{ f, n_1, ... \}
\]

\[
F[(if (p n_1 ...) c_t c_f)] = \{ n_1, ... \}
\]

Note: CPS/L₃ scoping rules ensure that continuation variables are never free in a function, so we ignore them.
To simplify some of the following slides, we assume that integer literals can be used as arguments of primitives. For example, we write:

\[
(\texttt{let}_p ((n (\text{block-get} b \ 1))) \ldots)
\]

instead of:

\[
(\texttt{let}^* ((c1 \ 1)
\begin{align*}
&\quad (n (\text{block-get} b c1)) \\
&\quad \ldots
\end{align*}
\)\]
Function definition

\[
[(\text{let}_f ((f_1 \ (\text{fun} \ (c_1 \ n_1,1 \ ...) \ e_1)) \ ...) \ e)] = \\
(\text{let}_f ((w_1 \ (\text{fun} \ (c_1 \ \text{env1} \ n_1,1 \ ...) \\
(\text{let}^* ((v_1 \ (\text{block-get} \ \text{env1} \ 1)) \\
...)) \\
[e_1][f_1 \rightarrow \text{env1}]\{\text{FV}_1(0) \rightarrow v_1\}[...])) \\
...)) \\
(\text{let}^* ((f_1 \ (\text{block-alloc-202} \ |\text{FV}_1|+1)) \\
...)) \\
(t_1 \ (\text{block-set!} \ f_1 \ 0 \ w_1)) \\
(t_2 \ (\text{block-set!} \ f_1 \ 1 \ \text{FV}_1(0))) \\
...)) \\
[e])
\]

\(\text{FV}_i = \text{an (arbitrary) ordering of the set } F[e_i] \setminus \{f_i, n_{i,1}, ... \}\)
Function application

Function application has to be transformed in order to extract the code pointer from the closure and pass the closure as the first argument after the return continuation:

\[ (\text{app}_f \ n \ n_c \ n_1 \ldots) = (\text{let}_p ((f \ (\text{block-get} \ n \ 0))) \ (\text{app}_f \ f \ n_c \ n \ n_1 \ldots)) \]
Function test

Functions being represented as tagged blocks, checking that an arbitrary object is a function amounts to checking that it is a tagged block and if it is, that its tag is 202. This can be done directly in L3, as a library function:

```lisp
(defun function? (fun o)
  (and (@block? o)
       (= 202 (@block-tag o)))))
```
Exercise

We have seen two techniques to represent the closures of mutually-recursive functions: cyclic closures and shared closures. Which of these two techniques does our transformation use (explain)?
Improving CPS/L_3 closure conversion
The translation just presented is suboptimal in two respects:

1. it always creates closures, even for functions that are never used as values (i.e. only applied),
2. it always performs calls through the closure, thereby making all calls indirect.

These problems could be solved by later optimizations or by a better version of the translation sketched below.
Translation inefficiencies

The inefficiency of the simple translation can be observed on the make-adder example:

```lisp
(def make-adder (fun (x)
             (fun (y) (@+ x y)))
(make-adder 1)
```

Applied to the CPS version of that program, the simple translation creates a closure for both functions. However, the outer one is closed and never used as a value, therefore no closure needs to be created for it.

Notice that even if the outer function escaped (i.e. was used as a value) and needed a closure, the call to it could avoid fetching its code pointer from the closure, as here it is a known function.
Improved translation

The simple translation translates a source function into one target function and one closure. The improved translation splits the target function in two:

1. the **wrapper**, which simply extracts the free variables from the environment and passes them as additional arguments to the worker,

2. the **worker**, which takes the free variables as additional arguments and does the real work.

The wrapper is put in the closure.
The worker is used directly whenever the source function is applied to arguments instead of being used as a value.
Improved function definition

\[
[(\text{let}_f \ ( ((f_1 \ (\text{fun} \ (c_1 \ n_{1,1} \ ... \ e_1)) \ ... \ e)) \ )] = \]

(\text{let}_f \ ( (w_1 \ (\text{fun} \ (c_1 \ n_{1,1} \ ... \ u_1 \ ...) \ [e_1]_{\{FV_1(0) \rightarrow u_1\}}\{\ldots\})))

(\text{s}_1 \ (\text{fun} \ (c_1 \ \text{env}_1 \ n_{11} \ ...)

\quad (\text{let}^* \ ( ((v_1 \ (\text{block-get} \ \text{env}_1 \ 1)) \ ...)

\quad \quad (\text{app}_f \ w_1 \ c_1 \ n_{11} \ ... \ v_1 \ ...))))

\quad ...)

\quad (\text{let}^* \ ( ((f_1 \ (\text{block-alloc-202} \ |FV_1|+1))

\quad \quad ...)

\quad \quad (t_1 \ (\text{block-set!} \ f_1 \ 0 \ s_1))

\quad \quad ...)

\quad [e])}
Improved function application

When translating function application, if the function being applied is known (i.e. is bound by an enclosing `let f`), the worker of that function can be used directly, without going through the closure:

\[
\boxed{\langle \text{app}_f \ n \ n_c \ n_1 \ldots \rangle} = \text{if } n \text{ is known, with worker } n_w \\
(\text{app}_f \ n_w \ n_c \ n_1 \ldots \ \text{FV}_n(0)\ldots)
\]

otherwise, the closure has to be used, as in the simple translation:

\[
\boxed{\langle \text{app}_f \ n \ n_c \ n_1 \ldots \rangle} = \text{otherwise} \\
(\text{let}_p \ ((f \ (\text{block-get} \ n \ 0))) \\
(\text{app}_f \ f \ n_c \ n \ n_1\ldots))
\]
Free variables

The improved translation makes the computation of free variables slightly more difficult. That's because when a function \( f \) calls a known function \( g \), it has to pass it its free variables as additional arguments. The free variables of \( g \) now become free variables of \( f \). These new free variables must be added to \( f \)'s arguments, which impacts its callers – which could include \( g \) if the two are mutually-recursive. And so on...
Hoisting
CPS/L$_3$ functions
After closure conversion, all functions in the program are closed. Therefore, it is possible to hoist them all to a single outer $\texttt{let}_f$.

Once this is done, the program has the following simple form:

$$\texttt{(let}_f \ (\textit{all functions of the program})
\text{}\textit{main program code})$$

where the main program code does not contain any function definition ($\texttt{let}_f$ expression).

Hoisting functions to the top level simplifies the shape of the program and can make the job of later phases – e.g. assembly code generation – easier.
CPS/L₃ hoisting (1)

\[ ([\text{let}_l ((n \, l)) \, e]) = \]
\[
(\text{let}_f (fs) \\
\quad (\text{let}_l ((n \, l)) \, e'))
\]
\[
\text{if}[e] = (\text{let}_f (fs) \, e')
\]

\[ ([\text{let}_p ((n \, (p \, n_1 \, ...))) \, e]) = \]
\[
(\text{let}_f (fs) \\
\quad (\text{let}_p ((n \, (p \, n_1 \, ...))) \, e'))
\]
\[
\text{if}[e] = (\text{let}_f (fs) \, e')
\]
CPS/L₃ hoisting (2)

\[ [(\text{let}_c ((c_1 (\text{cnt } (n_{1,1} \ldots) \ e_1)) \ldots) \ e)] = \\
(\text{let}_f (f_{s_1} \ldots f_{s})) \\
(\text{let}_c ((c_1 (\text{cnt } (n_{1,1} \ldots) \ e_{1}'))) e')) \\
if [e_i] = (\text{let}_f (f_{s_i}) e'_i) \\
and [e] = (\text{let}_f (f) e') \]

\[ [(\text{let}_f ((f_1 (\text{fun } (n_{1,1} \ldots) \ e_1)) \ldots) \ e)] = \\
(\text{let}_f ((f_1 (\text{fun } (n_{1,1} \ldots) \ e_1'))) \ldots f_{s_1} \ldots f_{s}) e') \\
if [e_i] = (\text{let}_f (f_{s_i}) e'_i) \\
and [e] = (\text{let}_f (f) e') \]

\[ [e] \text{ when } e \text{ is any other kind of expression} = \\
(\text{let}_f () \ e) \]
Closures and objects
Closures and objects

There is a strong similarity between closures and objects: closures can be seen as objects with a single method – containing the code of the closure – and a set of fields – the environment.

Anonymous nested classes can therefore be used to simulate closures, but the syntax for them is often too heavyweight to be used often.

In languages like Scala or Java 8, a special syntax exists for anonymous functions, which are translated to nested classes.
Adder maker in Scala

To see how closures are handled in Scala, let's look at how the translation of the Scala equivalent of the `make-adder` function:

```scala
def makeAdder(x: Int): Int => Int =
  { y: Int => x+y }
val increment = makeAdder(1)
increment(41)
```
Translated adder

(Hoisted) closure class: the code is in the apply method, the environment in the object itself: it's a flat closure.

class Anon extends Function1[Int,Int] {
  private val x: Int;
  def this(x: Int) = {
    this.x = x
  }
  def apply(y: Int): Int = this.x + y
}

def makeAdder(x: Int): Function1[Int,Int] =
  new Anon(x)
val increment = makeAdder(1)
increment.apply(41)