Functions and Data
In this section, we’ll learn how functions create and encapsulate data structures.

**Example**

Rational Numbers

We want to design a package for doing rational arithmetic.

A rational number \( \frac{x}{y} \) is represented by two integers:

- its *numerator* \( x \), and
- its *denominator* \( y \).
Rational Addition

Suppose we want to implement the addition of two rational numbers.

```python
def addRationalNumerator(n1: Int, d1: Int, n2: Int, d2: Int): Int
def addRationalDenominator(n1: Int, d1: Int, n2: Int, d2: Int): Int
```

but it would be difficult to manage all these numerators and denominators.

A better choice is to combine the numerator and denominator of a rational number in a data structure.
In Scala, we do this by defining a `class`:

```scala
class Rational(x: Int, y: Int) {
    def numer = x
    def denom = y
}
```

This definition introduces two entities:

- A new `type`, named `Rational`.
- A `constructor` `Rational` to create elements of this type.

Scala keeps the names of types and values in different namespaces. So there’s no conflict between the two definitions of `Rational`. 
Objects

We call the elements of a class type *objects*.

We create an object by prefixing an application of the constructor of the class with the operator `new`.

**Example**

```cpp
new Rational(1, 2)
```
Members of an Object

Objects of the class `Rational` have two `members`, `numer` and `denom`.

We select the members of an object with the infix operator `.` (like in Java).

**Example**

```scala
def main() = { 
  val x = new Rational(1, 2) 
  x.numer > 1 
  x.denom > 2 
}
```
Rational Arithmetic

We can now define the arithmetic functions that implement the standard rules.

\[
\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_1 d_2 + n_2 d_1}{d_1 d_2}
\]

\[
\frac{n_1}{d_1} - \frac{n_2}{d_2} = \frac{n_1 d_2 - n_2 d_1}{d_1 d_2}
\]

\[
\frac{n_1}{d_1} \cdot \frac{n_2}{d_2} = \frac{n_1 n_2}{d_1 d_2}
\]

\[
\frac{n_1}{d_1} / \frac{n_2}{d_2} = \frac{n_1 d_2}{d_1 n_2}
\]

\[
\frac{n_1}{d_1} = \frac{n_2}{d_2} \text{ iff } n_1 d_2 = d_1 n_2
\]
Implementing Rational Arithmetic

def addRational(r: Rational, s: Rational): Rational =
    new Rational(
        r.numer * s.denom + s.numer * r.denom,
        r.denom * s.denom)

def makeString(r: Rational) =
    r.numer + "/" + r.denom

makeString(addRational(new Rational(1, 2), new Rational(2, 3)))  > 7/6
Methods

One can go further and also package functions operating on a data abstraction in the data abstraction itself.

Such functions are called *methods*.

**Example**

Rational numbers now would have, in addition to the functions `numer` and `denom`, the functions `add`, `sub`, `mul`, `div`, `equal`, `toString`. 
Methods for Rationals

Here's a possible implementation:

```scala
class Rational(x: Int, y: Int) {
  def numer = x
  def denom = y
  def add(r: Rational) =
    new Rational(numer * r.denom + r.numer * denom, denom * r.denom)
  def mul(r: Rational) = ...
  ...
  override def toString = numer + "/" + denom
}
```

**Remark:** the modifier `override` declares that `toString` redefines a method that already exists (in the class `java.lang.Object`).
Calling Methods

Here is how one might use the new `Rational` abstraction:

```scala
val x = new Rational(1, 3)
val y = new Rational(5, 7)
val z = new Rational(3, 2)
x.add(y).mul(z)
```
Exercise

1. In your worksheet, add a method neg to class Rational that is used like this:

   x.neg // evaluates to -x

2. Add a method sub to subtract two rational numbers.

3. With the values of x, y, z as given in the previous slide, what is the result of

   x - y - z

   ?