Class Hierarchies
Consider the task of writing a class for sets of integers with the following operations.

```java
abstract class IntSet {
    def incl(x: Int): IntSet
    def contains(x: Int): Boolean
}
```

IntSet is an *abstract class*.

Abstract classes can contain members which are missing an implementation (in our case, incl and contains).

Consequently, no instances of an abstract class can be created with the operator `new`. 
Let’s consider implementing sets as binary trees.

There are two types of possible trees: a tree for the empty set, and a tree consisting of an integer and two sub-trees.

Here are their implementations:

```scala
class Empty extends IntSet {
  def contains(x: Int): Boolean = false
  def incl(x: Int): IntSet = new NonEmpty(x, new Empty, new Empty)
}
```
class NonEmpty(elem: Int, left: IntSet, right: IntSet) extends IntSet {

    def contains(x: Int): Boolean =
        if (x < elem) left contains x
        else if (x > elem) right contains x
        else true

    def incl(x: Int): IntSet =
        if (x < elem) new NonEmpty(elem, left incl x, right)
        else if (x > elem) new NonEmpty(elem, left, right incl x)
        else this
}
Empty and NonEmpty both *extend* the class IntSet.

This implies that the types Empty and NonEmpty *conform* to the type IntSet:

- an object of type Empty or NonEmpty can be used wherever an object of type IntSet is required.
IntSet is called the *superclass* of Empty and NonEmpty.

Empty and NonEmpty are *subclasses* of IntSet.

In Scala, any user-defined class extends another class.

If no superclass is given, the standard class `Object` in the Java package `java.lang` is assumed.

The direct or indirect superclasses of a class C are called *base classes* of C.

So, the base classes of NonEmpty are IntSet and Object.
Implementation and Overriding

The definitions of contains and incl in the classes Empty and NonEmpty implement the abstract functions in the base trait IntSet.

It is also possible to redefine an existing, non-abstract definition in a subclass by using override.

Example

```scala
abstract class Base {
  def foo = 1
  def bar: Int
}

class Sub extends Base {
  override def foo = 2
  def bar = 3
}
```
Object Definitions

In the IntSet example, one could argue that there is really only a single empty IntSet.

So it seems overkill to have the user create many instances of it.

We can express this case better with an object definition:

```object
object Empty extends IntSet {
    def contains(x: Int): Boolean = false
    def incl(x: Int): IntSet = new NonEmpty(x, Empty, Empty)
}
```

This defines a singleton object named Empty.

No other Empty instances can be (or need to be) created.

Singleton objects are values, so Empty evaluates to itself.
So far we have executed all Scala code from the REPL or the worksheet.

But it is also possible to create standalone applications in Scala. Each such application contains an object with a `main` method. For instance, here is the “Hello World!” program in Scala.

```scala
object Hello {
  def main(args: Array[String]) = println("hello world!")
}
```

Once this program is compiled, you can start it from the command line with

```
> scala Hello
```
Exercise

Write a method `union` for forming the union of two sets. You should implement the following abstract class.

```scala
abstract class IntSet {
    def incl(x: Int): IntSet
    def contains(x: Int): Boolean
    def union(other: IntSet): IntSet
}
```
Dynamic Binding

Object-oriented languages (including Scala) implement dynamic method dispatch.

This means that the code invoked by a method call depends on the runtime type of the object that contains the method.

Example

Empty contains 1
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**Example**

Empty contains 1

\[ [1/x] [Empty/this] \text{false} \]
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**Example**

Empty contains 1

\[
\rightarrow [1/x] [Empty/this] \text{false}
\]

= false
Another evaluation using NonEmpty:

(new NonEmpty(7, Empty, Empty)) contains 7
Dynamic Binding (2)

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(new NonEmpty(7, Empty, Empty)) contains 7

→ [7/\text{elem}] [7/\text{x}] [\text{new NonEmpty}(7, \text{Empty}, \text{Empty})/\text{this}]
  if (x < elem) this.left contains x
  else if (x > elem) this.right contains x else true
Dynamic Binding (2)

Another evaluation using NonEmpty:

(new NonEmpty(7, Empty, Empty)) contains 7

→ [7/\text{elem}] [7/x] [\text{new NonEmpty}(7,\text{Empty},\text{Empty})/\text{this}]  
  \text{if} (x < \text{elem}) \text{this.left contains } x  
  \text{else if} (x > \text{elem}) \text{this.right contains } x \text{ else true}

= \text{if} (7 < 7) \text{new NonEmpty}(7,\text{Empty},\text{Empty}).\text{left contains } 7  
  \text{else if} (7 > 7) \text{new NonEmpty}(7,\text{Empty},\text{Empty}).\text{right contains } 7 \text{ else true}
Another evaluation using NonEmpty:

(new NonEmpty(7, Empty, Empty)) contains 7

→ [7/elem] [7/x] [new NonEmpty(7,Empty,Empty)/this]
   if (x < elem) this.left contains x
   else if (x > elem) this.right contains x else true

= if (7 < 7) new NonEmpty(7, Empty, Empty).left contains 7
   else if (7 > 7) new NonEmpty(7, Empty, Empty).right
           contains 7 else true

→ true
Dynamic dispatch of methods is analogous to calls to higher-order functions.

*Question:*

Can we implement one concept in terms of the other?

- Objects in terms of higher-order functions?
- Higher-order functions in terms of objects?