Computing with Infinite Sequences
Infinite Streams

You saw that all elements of a stream except the first one are computed only when they are needed to produce a result.

This opens up the possibility to define infinite streams!

For instance, here is the stream of all integers starting from a given number:

    def from(n: Int): Stream[Int] = n #: from(n + 1)

The stream of all natural numbers:
Infinite Streams

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```scala
val nats = from(0)
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The stream of all multiples of 4:

```scala
nats map (_ * 4)
```
The Sieve of Eratosthenes is an ancient technique to calculate prime numbers.

The idea is as follows:

- Start with all integers from 2, the first prime number.
- Eliminate all multiples of 2.
- The first element of the resulting list is 3, a prime number.
- Eliminate all multiples of 3.
- Iterate forever. At each step, the first number in the list is a prime number and we eliminate all its multiples.
The Sieve of Eratosthenes in Code

Here's a function that implements this principle:

```scala
def sieve(s: Stream[Int]): Stream[Int] =  
  s.head #:: sieve(s.tail filter (_ % s.head != 0))

val primes = sieve(from(2))

To see the list of the first N prime numbers, you can write

(primes take N).toList
```
Our previous algorithm for square roots always used a `isGoodEnough` test to tell when to terminate the iteration.

With streams we can now express the concept of a converging sequence without having to worry about when to terminate it:

```scala
def sqrtStream(x: Double): Stream[Double] = {
  def improve(guess: Double) = (guess + x / guess) / 2
  lazy val guesses: Stream[Double] = 1 #:: (guesses map improve)
  guesses
}
```
We can add isGoodEnough later.

```scala
def isGoodEnough(guess: Double, x: Double) =
  math.abs((guess * guess - x) / x) < 0.0001

sqrtStream(4) filter (isGoodEnough(_, 4))
```
Exercise:

Consider two ways to express the infinite stream of multiples of a given number N:

\[
\text{val } xs = \text{from}(1) \text{ map } (_ \times N)
\]

\[
\text{val } ys = \text{from}(1) \text{ filter } (_ \mod N == 0)
\]

Which of the two streams generates its results faster?

0 \text{ from}(1) \text{ map } (_ \times N)
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Exercise:

Consider two ways to express the infinite stream of multiples of a given number \( N \):

```scala
type N = 3

val xs = from(1) map (_ * N)

val ys = from(1) filter (_ % N == 0)
```

Which of the two streams generates its results faster?

- \( \text{from}(1) \text{ map } (\_ \ast N) \)
- \( \text{from}(1) \text{ filter } (\_ \% N == 0) \)