

# Register allocation

Advanced Compiler Construction

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# Register allocation

The problem of **register allocation** consists in rewriting a program that makes use of an unbounded number of local variables – also called **virtual** or **pseudo-registers** – into one that only makes use of machine registers.

If there are not enough machine registers to store all variables, one or several of them must be **spilled**, i.e. stored in memory instead of in a register.

Register allocation is one of the very last phases of the compilation process – typically only instruction scheduling comes later. It is performed on an intermediate language that is extremely close to machine code.

# Setting the scene

Register allocation will be performed on an RTL language with the following characteristics:

- apart from  $n$  machine registers  $R_0, \dots, R_{n-1}$ , an unbounded number of virtual registers  $v_0, v_1, \dots$  are available before register allocation,
- machine registers that play a special role, like the link register (containing the return address), are identified with a non-numerical index, e.g.  $R_{LK}$ ; they are real registers nevertheless.

# Running example

To illustrate register allocation techniques, we will use a function computing the greatest common divisor of two numbers using Euclid's algorithm.

## In L<sub>3</sub>

```
(defrec gcd
  (fun (a b)
    (if (= 0 b)
        a
        (gcd b (% a b)))))
```

## In RTL

```
gcd:  R3 ← done
      if R2 = 0 goto R3
      R3 ← R2
      R2 ← R1 % R2
      R1 ← R3
      R3 ← gcd
      goto R3
done: goto RLK
```

Calling conventions: arguments are passed in R<sub>1</sub>, R<sub>2</sub>, ... and the result is put in R<sub>1</sub>.

# Register allocation example

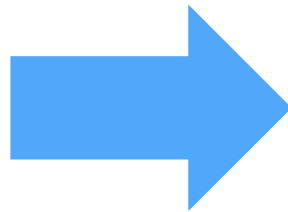
## Before register allocation

```
gcd:  v0 ← RLK
      v1 ← R1
      v2 ← R2
loop: v3 ← done
      if v2 = 0 goto v3
      v4 ← v2
      v2 ← v1 % v2
      v1 ← v4
      v5 ← loop
      goto v5
done: R1 ← v1
      goto v0
```

R<sub>1</sub>, R<sub>2</sub>: parameters

R<sub>LK</sub>: return address

allocable  
registers:  
R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>,  
R<sub>LK</sub>



## After register allocation

```
gcd:
loop: R3 ← done
      if R2 = 0 goto R3
      R3 ← R2
      R2 ← R1 % R2
      R1 ← R3
      R3 ← loop
      goto R3
done: goto RLK
```

Allocation:

v<sub>0</sub> → R<sub>LK</sub>

v<sub>1</sub> → R<sub>1</sub>

v<sub>2</sub> → R<sub>2</sub>

v<sub>3</sub>, v<sub>4</sub>, v<sub>5</sub> → R<sub>3</sub>

# Techniques

We will study two commonly used techniques:

- register allocation by **graph coloring**, which is relatively slow but produces very good results,
- **linear scan** register allocation, which is fast but produces worse results.

Because it is slow, graph coloring tends to be used in batch compilers, while linear scan tends to be used in JIT compilers.

Both techniques are **global**, i.e. they allocate registers for a whole function at a time.

# Technique #1: graph coloring

# Allocation by graph coloring

The problem of register allocation can be reduced to the well-known problem of graph coloring, as follows:

- The **interference graph** is built. It has one node per register – real or virtual – and two nodes are connected by an edge iff their registers are simultaneously live.
- The interference graph is colored with at most  $K$  colors ( $K$  being the number of available registers) so that all nodes have a different color than all their neighbors.

Problems:

- for an arbitrary graph, the coloring problem is NP-complete,
- a  $K$ -coloring might not even exist.



# Interference graph example

## Program

```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2=0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

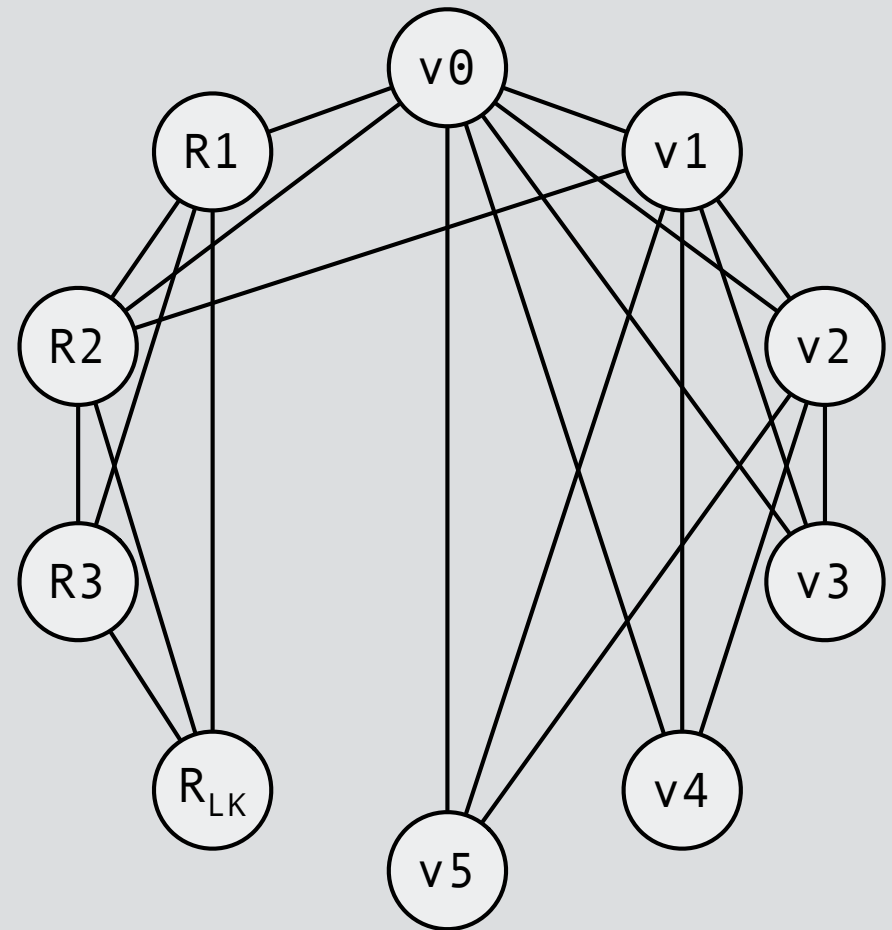
## Liveness

{in}{out}

```
{R1,R2,RLK}{R1,R2,v0}
{R1,R2,v0}{R2,v0,v1}
{R2,v0,v1}{v0-v2}
{v0-v2}{v0-v3}
{v0-v3}{v0-v2}
{v0-v2}{v0-v2,v4}
{v0-v2,v4}{v0-v2,v4}
{v0-v2,v4}{v0-v2}
{v0-v2}{v0-v2,v5}
{v0-v2,v5}{v0-v2}

{v0,v1}{R1,v0}
{R1,v0}{R1}
```

## Interference graph

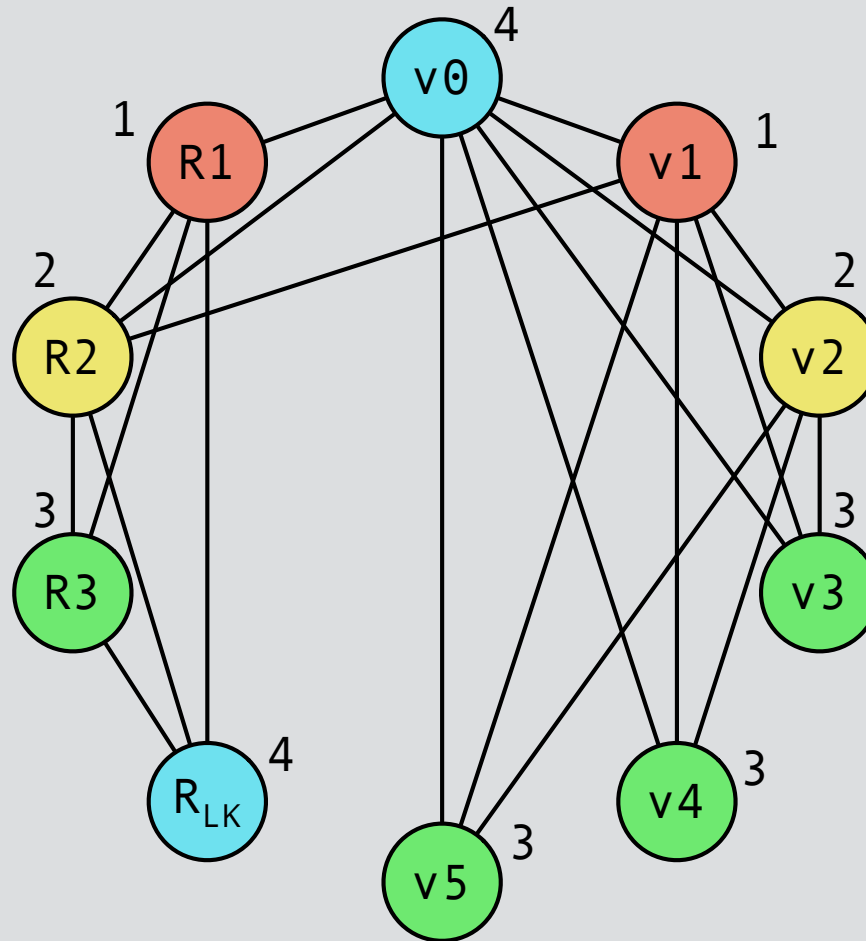


# Coloring example

## Original prog.

```
gcd:  
  v0 ← RLK  
  v1 ← R1  
  v2 ← R2  
loop:  
  v3 ← done  
  if v2=0 goto v3  
  v4 ← v2  
  v2 ← v1 % v2  
  v1 ← v4  
  v5 ← loop  
  goto v5  
done:  
  R1 ← v1  
  goto v0
```

## Colored interference graph



## Rewritten prog.

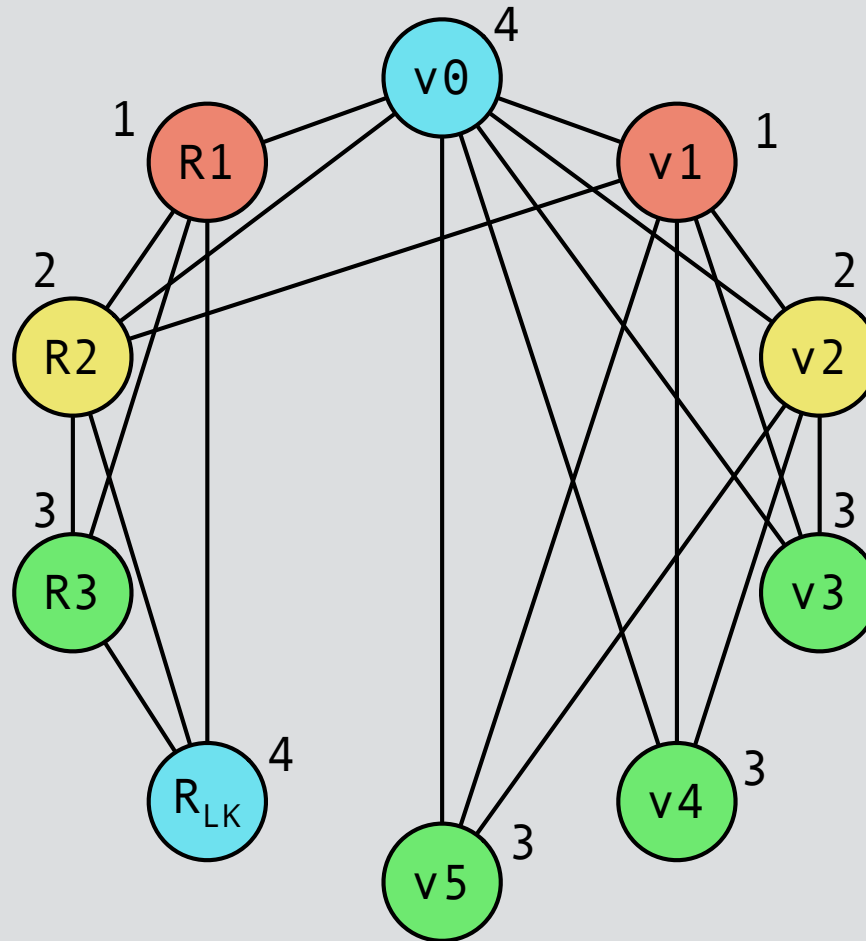
```
gcd:  
  RLK ← RLK  
  R1 ← R1  
  R2 ← R2  
loop:  
  R3 ← done  
  if R2=0 goto R3  
  R3 ← R2  
  R2 ← R1 % R2  
  R1 ← R3  
  R3 ← loop  
  goto R3  
done:  
  R1 ← R1  
  goto RLK
```

# Coloring example

## Original prog.

```
gcd:  
  v0 ← RLK  
  v1 ← R1  
  v2 ← R2  
loop:  
  v3 ← done  
  if v2=0 goto v3  
  v4 ← v2  
  v2 ← v1 % v2  
  v1 ← v4  
  v5 ← loop  
  goto v5  
done:  
  R1 ← v1  
  goto v0
```

## Colored interference graph



## Rewritten prog.

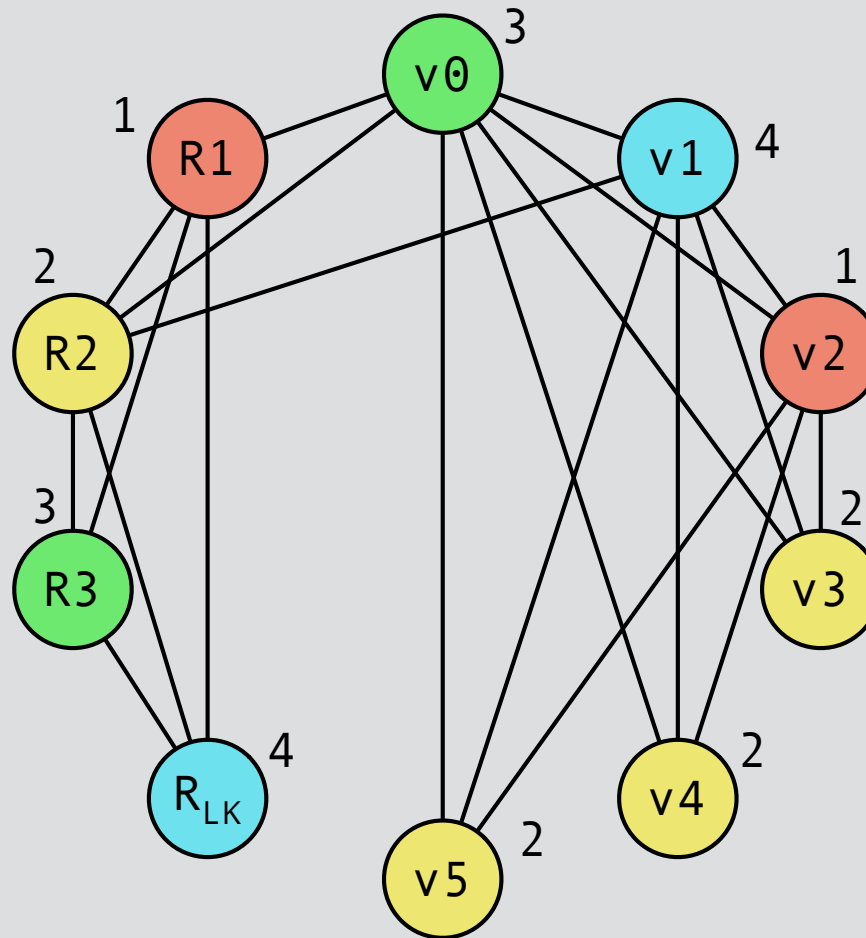
```
gcd:  
RLK ← RLK  
R1 ← R1  
R2 ← R2  
loop:  
  R3 ← done  
  if R2=0 goto R3  
  R3 ← R2  
  R2 ← R1 % R2  
  R1 ← R3  
  R3 ← loop  
  goto R3  
done:  
R1 ← R1  
  goto RLK
```

# Coloring example (2)

## Original prog.

```
gcd:  
  v0 ← RLK  
  v1 ← R1  
  v2 ← R2  
loop:  
  v3 ← done  
  if v2=0 goto v3  
  v4 ← v2  
  v2 ← v1 % v2  
  v1 ← v4  
  v5 ← loop  
  goto v5  
done:  
  R1 ← v1  
  goto v0
```

## Colored interference graph



## Rewritten prog.

```
gcd:  
  R3 ← RLK  
  RLK ← R1  
  R1 ← R2  
loop:  
  R2 ← done  
  if R1=0 goto R2  
  R2 ← R1  
  R1 ← RLK % R1  
  RLK ← R2  
  R2 ← loop  
  goto R2  
done:  
  R1 ← RLK  
  goto R3
```

This second coloring is also correct, but produces worse code!

# Coloring by simplification

**Coloring by simplification** is a heuristic technique to (try to) color a graph  $G$  with  $K$  colors.

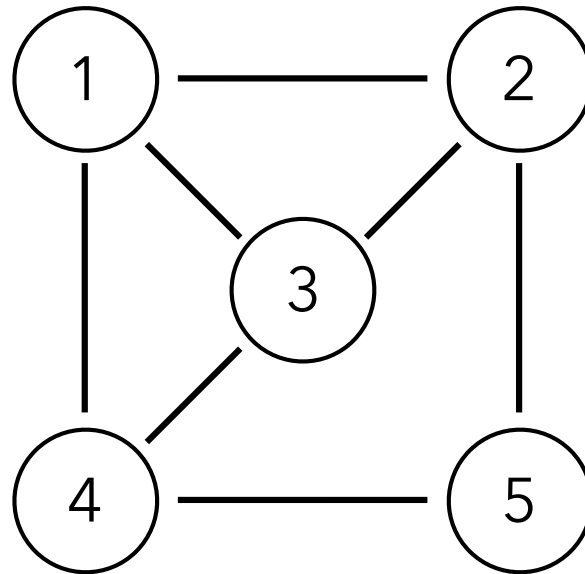
It works as follows: if the graph  $G$  has at least one node  $n$  with less than  $K$  neighbors,  $n$  is removed from  $G$ , and that simplified graph is recursively colored. Once this is done,  $n$  is colored with any color not used by its neighbors.

There is always at least one color available for  $n$ , because its neighbors use at most  $K-1$  colors.

If the graph does not contain a node with less than  $K$  neighbors,  $K$ -coloring might not be feasible, but will be attempted nevertheless, as we will see.

# Coloring by simplification

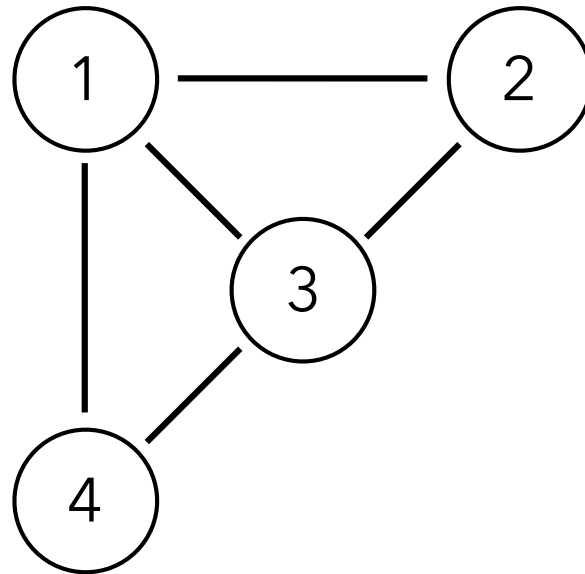
To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes:

# Coloring by simplification

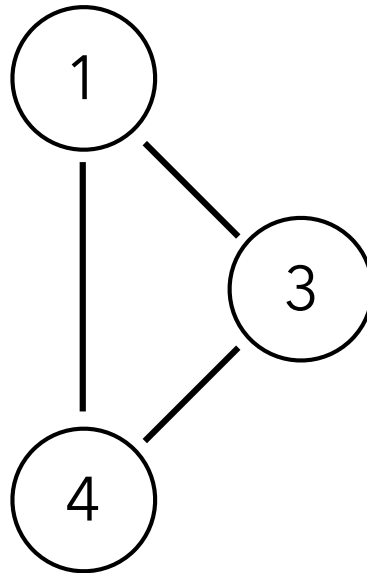
To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes: 5

# Coloring by simplification

To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.

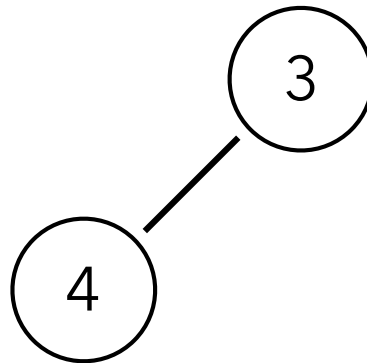


Stack of removed nodes: 5 2



# Coloring by simplification

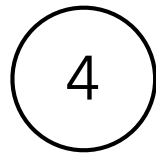
To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes: 5 2 1

# Coloring by simplification

To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes: 5 2 1 3

# Coloring by simplification

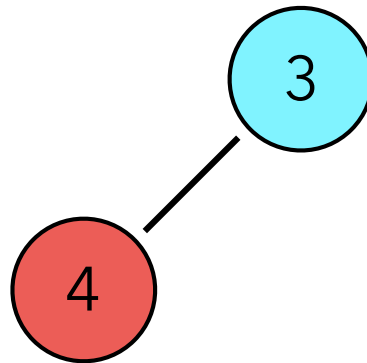
To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes: 5 2 1 3

# Coloring by simplification

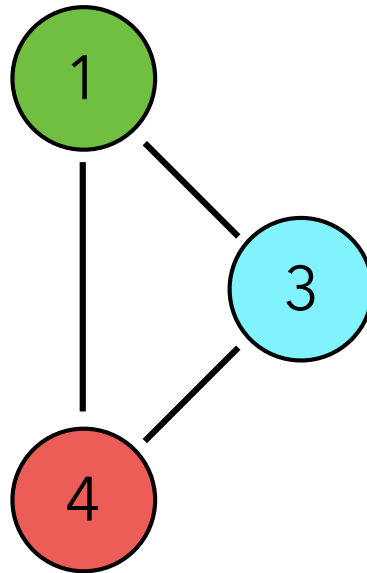
To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes: 5 2 1

# Coloring by simplification

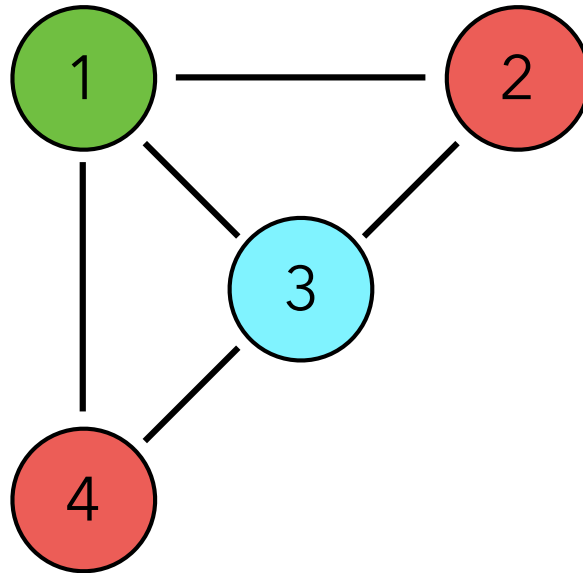
To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes: 5 2

# Coloring by simplification

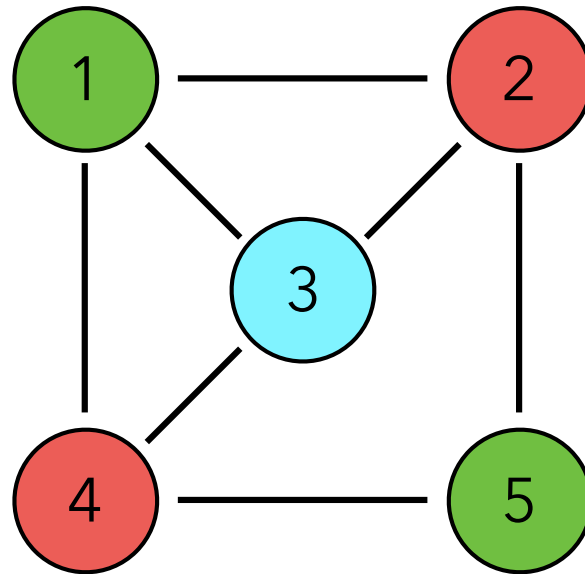
To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes: 5

# Coloring by simplification

To illustrate coloring by simplification, we can color the following graph with  $K=3$  colors.



Stack of removed nodes:

**Spilling**



# (Optimistic) spilling

During simplification, it is perfectly possible to reach a point where all nodes have at least  $K$  neighbors.

When this occurs, a node  $n$  must be chosen to be **spilled**, i.e. have its value stored in memory instead of in a register.

As a first approximation, we assume that the spilled value does not interfere with any other value, remove its node from the graph, and recursively color the simplified graph as usual.

After the simplified graph has been colored, it is actually possible that the neighbors of  $n$  do not use all the possible colors! In this case,  $n$  is not spilled. Otherwise it must really be spilled.

# Spill costs

The node to spill could be chosen at random, but it is clearly better to favor values that are not frequently used, or values that interfere with many others.

The following formula is often used as a measure of the spill cost for a node  $n$ . The node with the lowest cost should be spilled first.

$$\text{cost}(n) = (rw_0(n) + 10 rw_1(n) + \dots + 10^k rw_k(n)) / \text{degree}(n)$$

where  $rw_i(n)$  is the number of times the value of  $n$  is read or written in a loop of depth  $i$ , and  $\text{degree}(n)$  is the number of edges adjacent to  $n$  in the interference graph.

# Spilling of pre-colored nodes

As we have seen, the interference graph contains nodes corresponding to the registers of the machine.

These nodes are said to be **pre-colored**, because the color of each of them is given by the machine register it represents.

Pre-colored nodes must never be simplified during the coloring process, as by definition they cannot be spilled.

# Spilling example

Let's try to color the same interference graph as before, but with only three colors. There is no node with degree less than three, so the one with the lowest cost must be spilled.

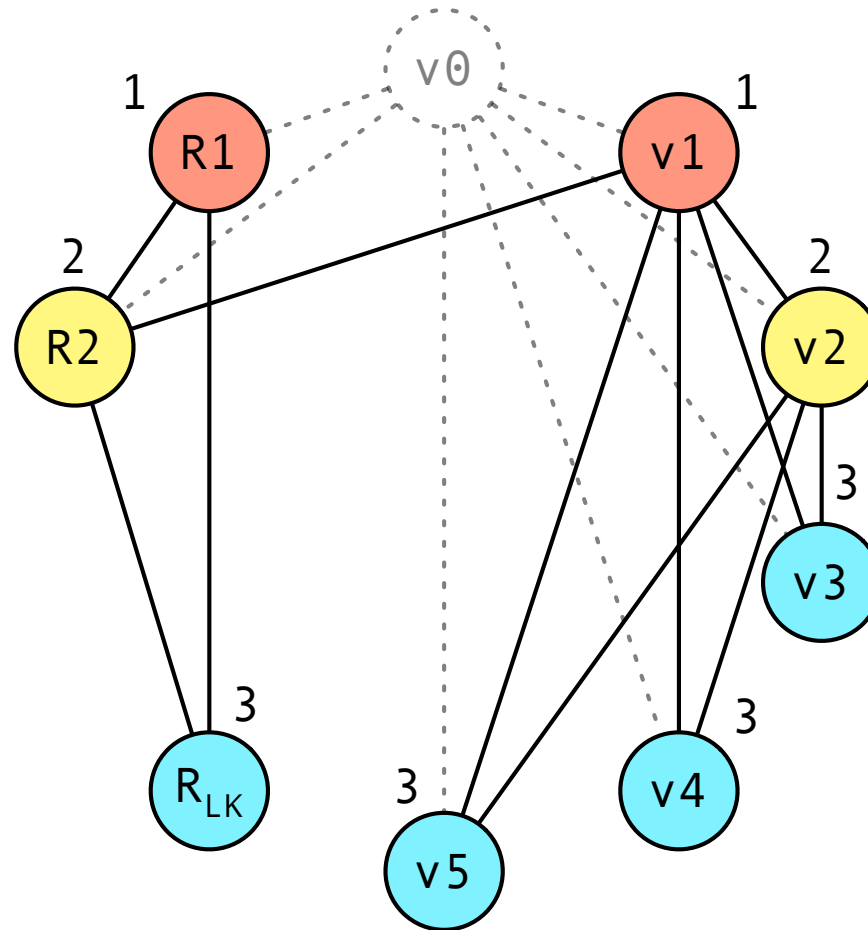
```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2=0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

node	rw <sub>0</sub>	rw <sub>1</sub>	deg.	cost
v <sub>0</sub>	2	0	7	0.29
v <sub>1</sub>	2	2	6	3.67
v <sub>2</sub>	1	4	6	6.83
v <sub>3</sub>	0	2	3	6.67
v <sub>4</sub>	0	2	3	6.67
v <sub>5</sub>	0	2	3	6.67

$$\text{cost} = (\text{rw}_0 + 10 \text{rw}_1) / \text{degree}$$

# Spilling example

Once  $v_0$ , which has the lowest spill cost, is removed from the graph, the simplified graph is 3-colorable.



# Consequences of spilling

Once a node has been spilled, the original program must be rewritten to take that spilling into account, as follows:

- just before the spilled value is read, code must be inserted to fetch it from memory,
- just after the spilled value is written, code must be inserted to write it back to memory.

Since that spilling code introduces new virtual registers, the whole register allocation process must be restarted from the beginning.

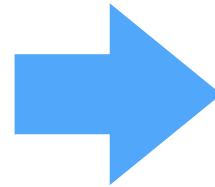
In practice, one or two iterations are enough in almost all cases.

# Spilling code integration

## Original program

```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2 = 0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

spilling  
of v<sub>0</sub>

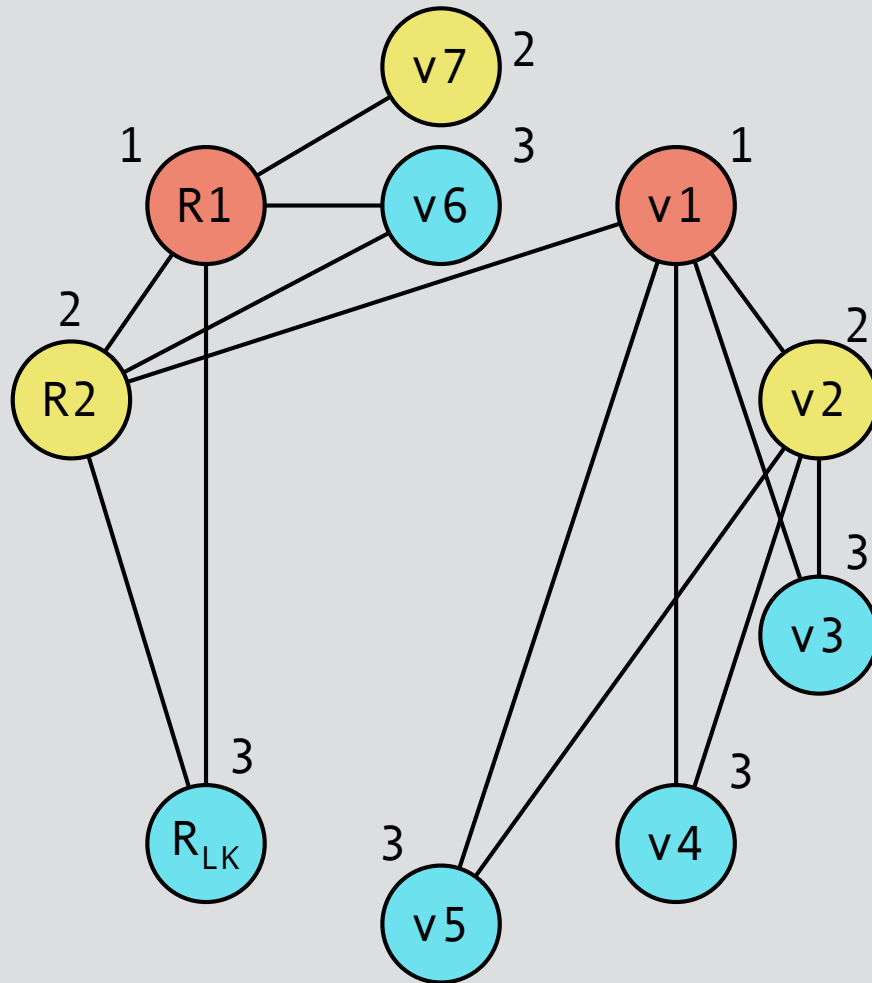


## Rewritten program

```
gcd:
  v6 ← RLK
  push v6
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2 = 0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  pop v7
  goto v7
```

# New interference graph

Interference graph w/ spilling



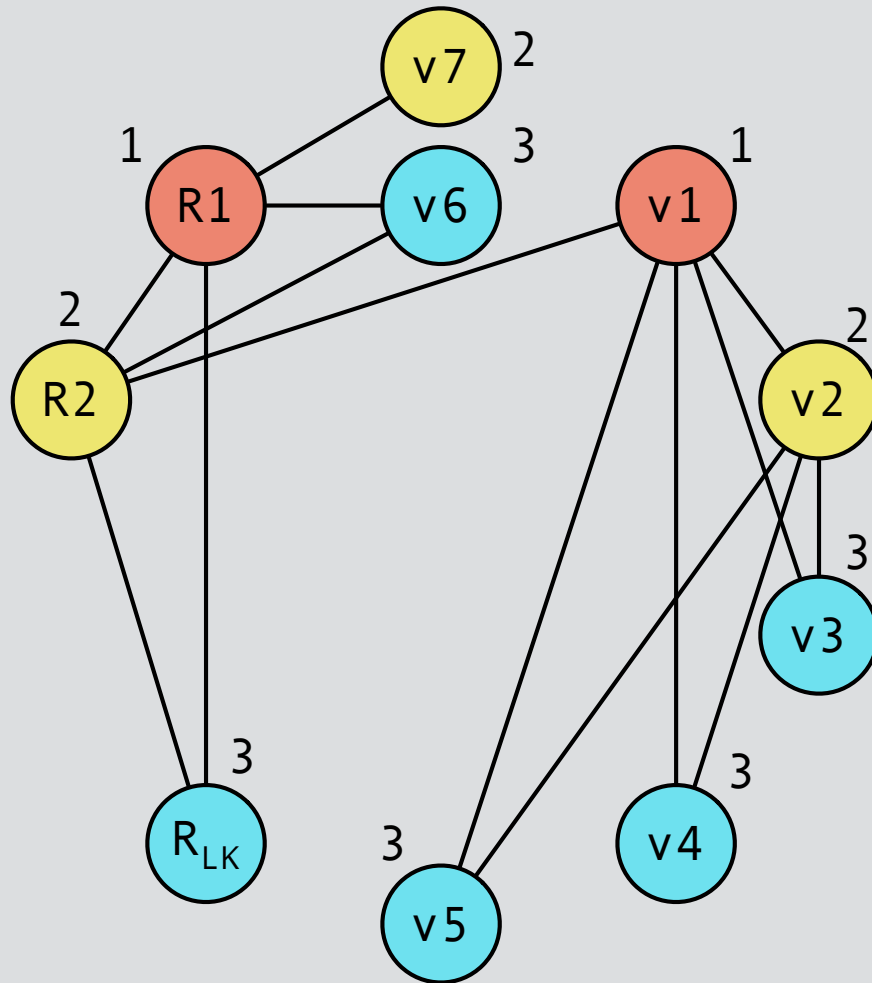
Final program

```
gcd:  
  RLK ← RLK  
  push RLK  
  R1 ← R1  
  R2 ← R2  
loop:  
  RLK ← done  
  if R2 = 0 goto RLK  
  RLK ← R2  
  R2 ← R1 % R2  
  R1 ← RLK  
  RLK ← loop  
  goto RLK  
done:  
  R1 ← R1  
  pop R2  
  goto R2
```



# New interference graph

Interference graph w/ spilling



Final program

gcd:

~~R<sub>LK</sub> ← R<sub>LK</sub>~~

push R<sub>LK</sub>

~~R<sub>1</sub> ← R<sub>1</sub>~~

~~R<sub>2</sub> ← R<sub>2</sub>~~

loop:

R<sub>LK</sub> ← done

if R<sub>2</sub> = 0 goto R<sub>LK</sub>

R<sub>LK</sub> ← R<sub>2</sub>

R<sub>2</sub> ← R<sub>1</sub> % R<sub>2</sub>

R<sub>1</sub> ← R<sub>LK</sub>

R<sub>LK</sub> ← loop

goto R<sub>LK</sub>

done:

R<sub>1</sub> ← R<sub>1</sub>

pop R<sub>2</sub>

goto R<sub>2</sub>

# Coalescing

# Coloring quality

As we have seen in our first example, two valid  $K$ -colorings of the same interference graph are not necessarily equivalent: one can lead to a much shorter program than the other.

This is due to the fact that a move instruction of the form

$$v_1 \leftarrow v_2$$

can be removed after register allocation if  $v_1$  and  $v_2$  end up being allocated to the same register. (Of course, this also holds when  $v_1$  or  $v_2$  is a real register before allocation).

A good register allocator must therefore try to make sure that this happens as often as possible.

# Coalescing

Given a move instruction of the form

$$v_1 \leftarrow v_2$$

and provided that  $v_1$  and  $v_2$  do not interfere, it is always possible to replace all instances of  $v_1$  and  $v_2$  by instances of a new virtual register  $v_{1\&2}$ . Once this has been done, the move instruction becomes useless and can be removed.

This technique is known as **coalescing**, as the nodes of  $v_1$  and  $v_2$  in the interference graph coalesce into a single node.

# Coalescing issue

Coalescing is not always a good idea, though: the coalesced node can have a higher degree than the two original nodes, which might make the graph impossible to color with  $K$  colors and require spilling!

Conservative coalescing heuristics have to be used.

# Coalescing heuristics

Two coalescing heuristics are commonly used:

**Briggs:** coalesce nodes  $n_1$  and  $n_2$  to  $n_{1\&2}$  iff  $n_{1\&2}$  has less than  $K$  neighbors of significant degree (i.e. of a degree greater or equal to  $K$ ),

**George:** coalesce nodes  $n_1$  and  $n_2$  to  $n_{1\&2}$  iff all neighbors of  $n_1$  either already interfere with  $n_2$  or are of insignificant degree.

Both heuristics are safe, in that they will not turn a  $K$ -colorable graph into a non- $K$ -colorable one. But they are also conservative, in that they might prevent a coalescing that would be safe.

# Heuristic #1: Briggs

Briggs' heuristic: coalesce nodes  $n_1$  and  $n_2$  to  $n_{1\&2}$  iff  $n_{1\&2}$  has less than  $K$  neighbors of significant degree (i.e. of degree  $\geq K$ ).

Rationale: during simplification, all the neighbors of  $n_{1\&2}$  that are of insignificant degree will be simplified; at this point,  $n_{1\&2}$  will have less than  $K$  neighbors and will therefore be simplifiable too.

This heuristic is safe, in that it will not turn a  $K$ -colorable graph into a non- $K$ -colorable one. But it is also conservative, in that it might prevent a coalescing that would be safe.

# Heuristic #2: George

George's heuristic: coalesce nodes  $n_1$  and  $n_2$  to  $n_{1\&2}$  iff all neighbors of  $n_1$  either already interfere with  $n_2$  or are of insignificant degree.

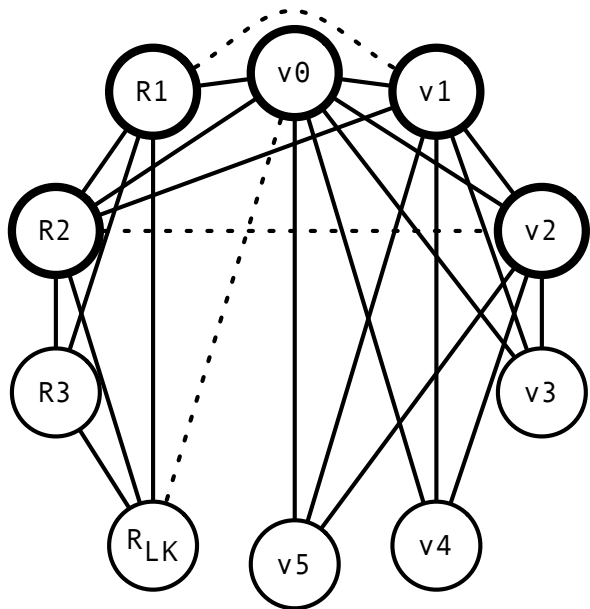
Rationale: the neighbors of  $n_{1\&2}$  will be the same as the neighbors of  $n_2$ , plus all neighbors of  $n_1$  that are of insignificant degree. The latter ones will all be simplified, at which point the graph will be a sub-graph of the original one.

Like Briggs', George's heuristic is safe but conservative.



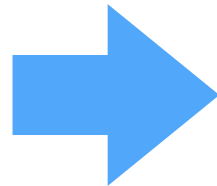
# Coalescing example

non-interfering, move-related nodes



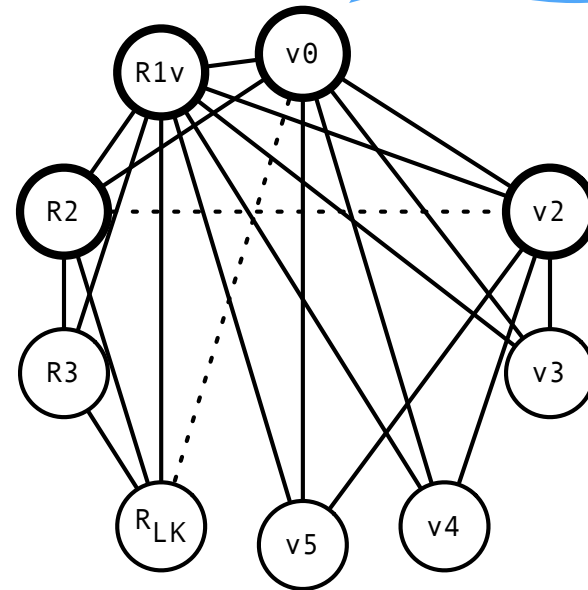
coalescing of  $R_1$  and  $v_1$  into

$R_{1v}$



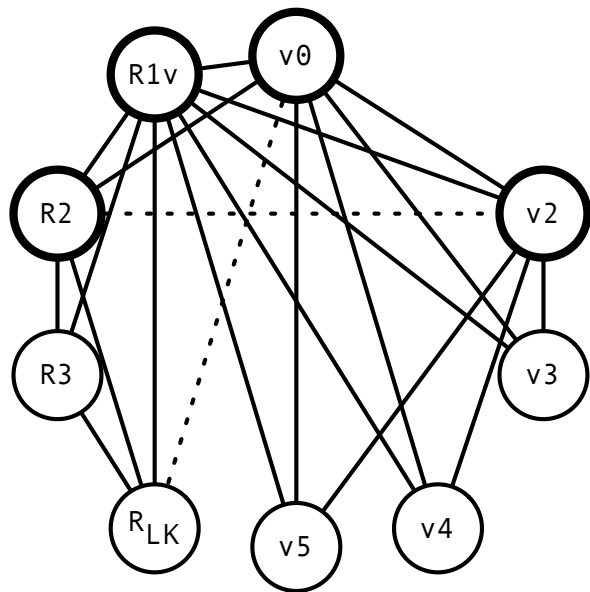
safe according to Briggs and George with  $K = 4$

node of significant degree



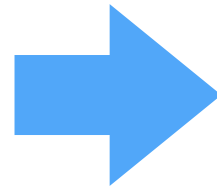
node of insignificant degree

# Coalescing example (2)

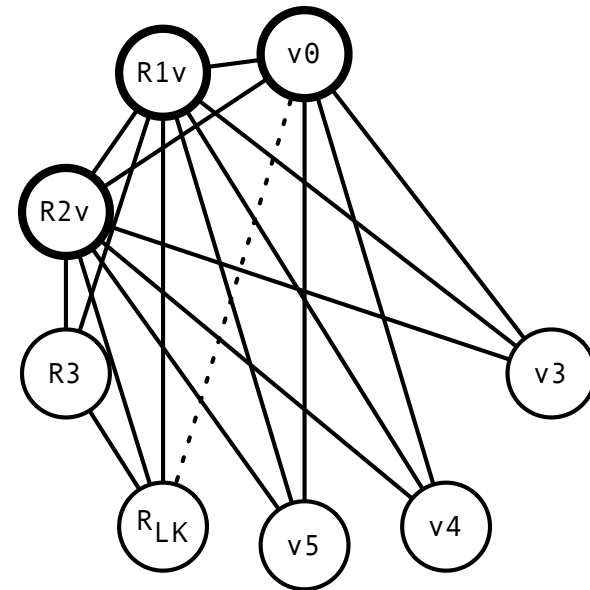


coalescing of  
 $R_2$  and  $v_2$  into

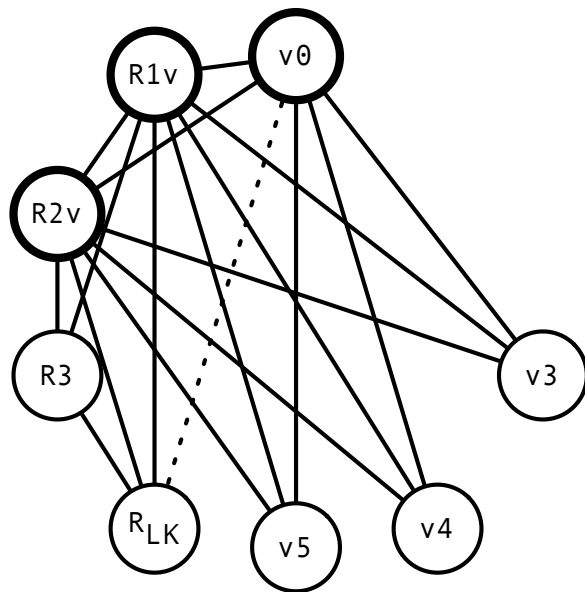
$R_{2v}$



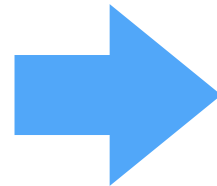
safe  
according to  
Briggs and  
George with  
 $K = 4$



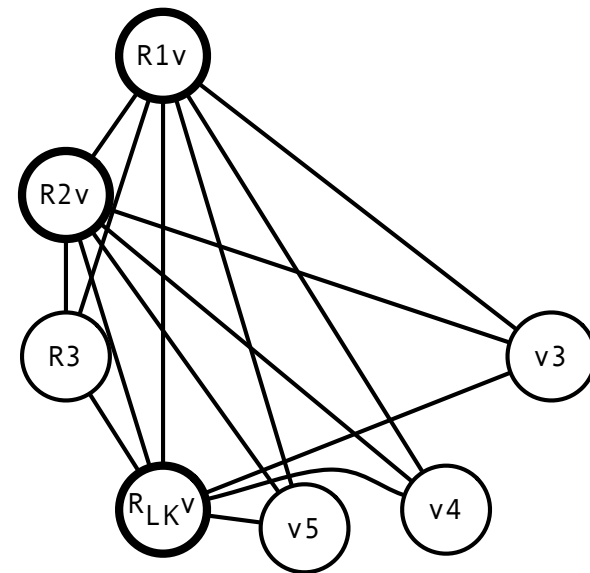
# Coalescing example (3)



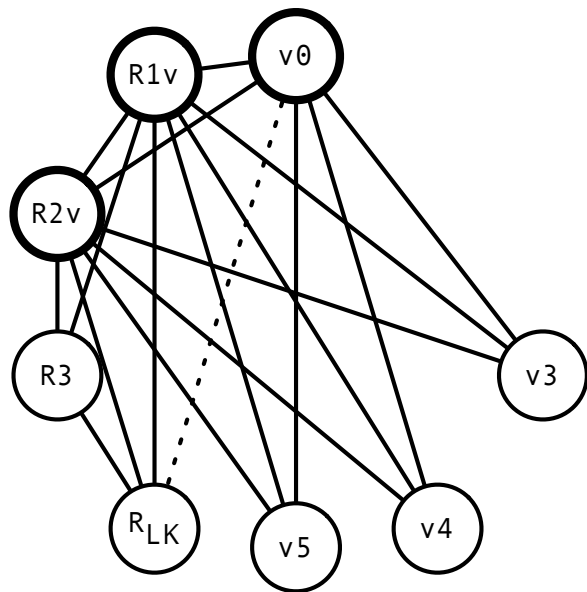
coalescing of  
 $R_{LK}$  and  $v_0$   
into  $R_{LKv}$



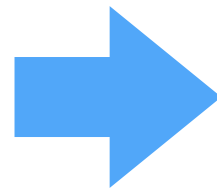
safe  
according to  
Briggs *and*  
George with  
 $K = 4$



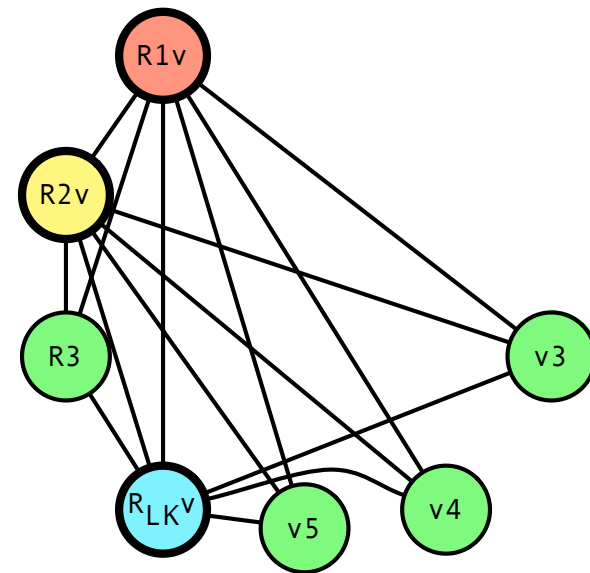
# Coalescing example (3)



coalescing of  
 $R_{LK}$  and  $v_0$   
into  $R_{LKv}$



safe  
according to  
Briggs *and*  
George with  
 $K = 4$



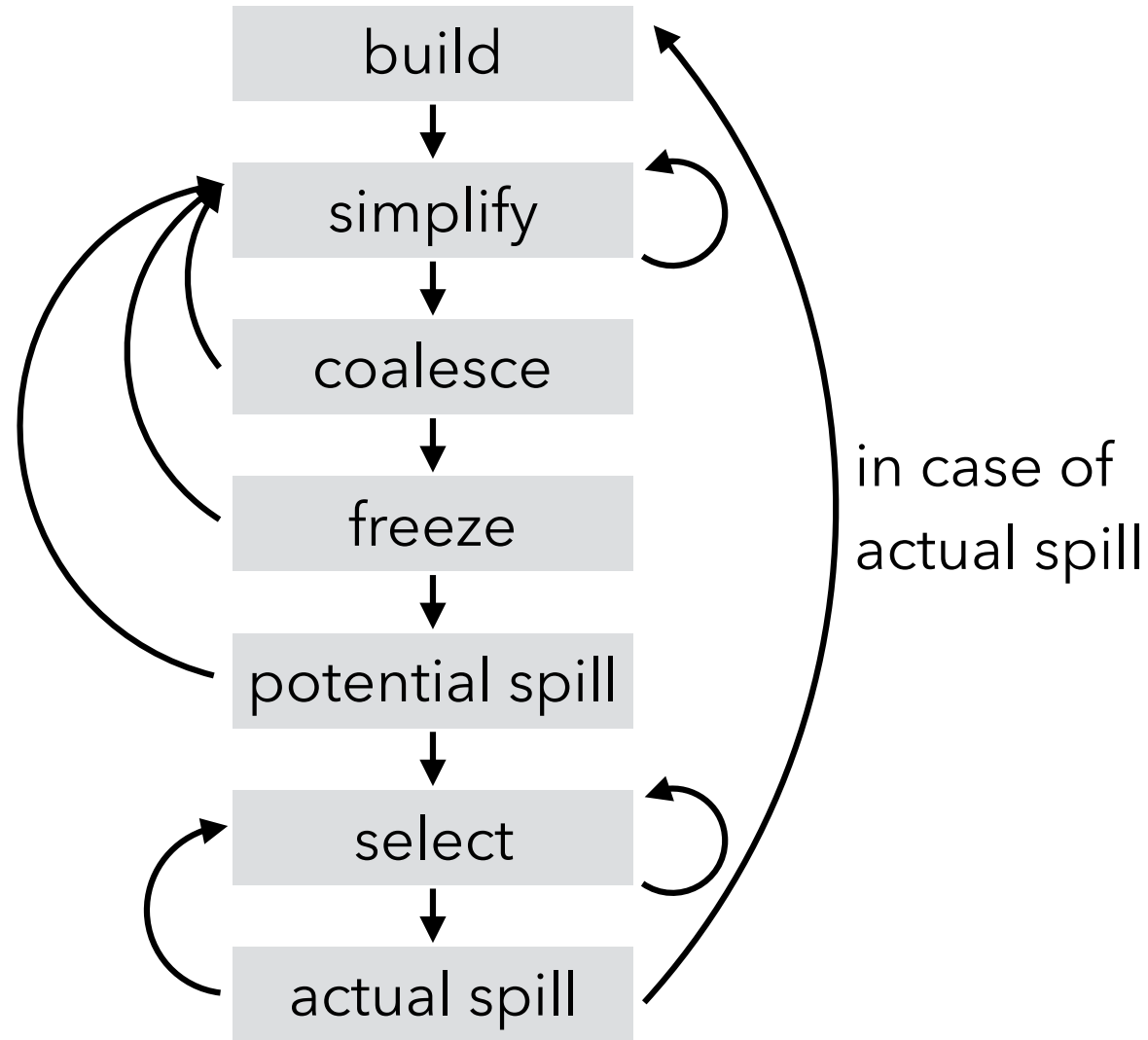
**Putting it all  
together**

# Iterated register coalescing

To get the best results, the phases of simplification and coalescing should be interleaved. This technique is known as **iterated register coalescing (IRC)**:

1. Interference graph nodes are partitioned in two classes: move-related or not.
2. Simplification is done on *not* move-related nodes (as move-related ones could be coalesced).
3. Conservative coalescing is performed.
4. When neither simplification nor coalescing can proceed further, some move-related nodes are **frozen** (marked as non-move-related).
5. The process is restarted at 2.

# Iterated register coalescing



# **Assignment constraints**



# Assignment constraints

Until now, we have assumed that a virtual register can be assigned to any physical register, as long as it is free.

In practice, this is often not the case, as various architectural characteristics impose **assignment constraints**, e.g.:

- some architectures divide the registers in several classes, with different capabilities (e.g. address vs. data registers, integer vs. floating-point registers, etc.),
- some instructions require some of their arguments – or their result – to be in specific registers,
- calling conventions require function arguments and results to be in specific registers.

A realistic register allocator has to be able to satisfy these constraints.

# Register classes

Most architectures separate the registers in several classes. Even in modern RISC architectures, there is typically one class for floating-point values and another one for integers and pointers.

Register classes can easily be taken into account in a coloring-based allocator: if a variable must be put in a register of some class, then its node can be made to interfere with all pre-colored nodes corresponding to registers of other classes.

# Calling conventions

Many calling conventions pass arguments in registers.

At the beginning of all functions, move instructions have to be inserted to copy the arguments to new virtual registers:

fact:

$v_1 \leftarrow R_1$  ; save first argument in  $v_1$

Similarly, before any function call, move instructions have to be inserted to load the arguments in the appropriate registers:

$R_1 \leftarrow v_2$  ; load first argument from  $v_2$

CALL fact

Whenever possible, these move instructions will be removed by coalescing.

# Caller/callee-saved registers

Calling conventions distinguish two kinds of registers:

- **caller-saved** registers are saved by the caller before a call and restored after it,
- **callee-saved** registers are saved by the callee at function entry and restored before function exit.

Ideally, all virtual registers that have to survive at least one call should be assigned to callee-saved registers, while other virtual registers should be assigned to caller-saved registers.

How can this be obtained in a coloring-based allocator?

# Caller/callee-saved registers

The contents of caller-saved registers do not survive a function call. To model this, edges are added to the interference graph between all virtual registers that are live across at least one call and (physical) caller-saved registers. These edges ensure that virtual registers that are live across at least one call will not be assigned to caller-saved registers, and will therefore either be spilled or allocated to callee-saved registers!

# Saving callee-saved registers

Callee-saved registers must be preserved by all functions. This can be achieved by copying them to fresh temporary registers at function entry and restoring them before exit.

# Saving callee-saved registers

For example, if  $R_8$  is a callee-saved register, a function could look like:

```
entry:  
     $v_1 \leftarrow R_8$     ; save callee-saved  $R_8$  in  $v_1$   
    ...                  ; function body  
     $R_8 \leftarrow v_1$     ; restore callee-saved  $R_8$   
    goto  $R_{LK}$ 
```

If the register pressure is low, then  $R_8$  and  $v_1$  will be coalesced, and the two move instructions removed. If register pressure is high,  $v_1$  will be spilled, thereby making  $R_8$  available in the function body, e.g. to store a virtual register live across a call.

# **Technique #2: linear scan**



# Linear scan

The basic linear scan technique is very simple:

- the program is linearized – i.e. represented as a linear sequence of instructions, not as a graph,
- a unique live range is computed for every variable, going from the first to the last instruction during which it is live,
- registers are allocated by iterating over the intervals sorted by increasing starting point: each time an interval starts, the next free register is allocated to it, and each time an interval ends, its register is freed,
- if no register is available, the active range ending last is chosen to have its variable spilled.

# Linear scan example

Let's try to allocate registers for our gcd procedure using linear scan, first with four allocable registers, then with three.

## Program

```
1 gcd:  v0 ← RLK
2      v1 ← R1
3      v2 ← R2
4 loop: v3 ← done
5      if v2=0 goto v3
6      v4 ← v2
7      v2 ← v1 % v2
8      v1 ← v4
9      v5 ← loop
10     goto v5
11 done: R1 ← v1
12     goto v0
```

## Live ranges

v<sub>0</sub>: [1<sup>+</sup>, 12<sup>-</sup>]

v<sub>1</sub>: [2<sup>+</sup>, 11<sup>-</sup>]

v<sub>2</sub>: [3<sup>+</sup>, 10<sup>+</sup>]

v<sub>3</sub>: [4<sup>+</sup>, 5<sup>-</sup>]

v<sub>4</sub>: [6<sup>+</sup>, 8<sup>-</sup>]

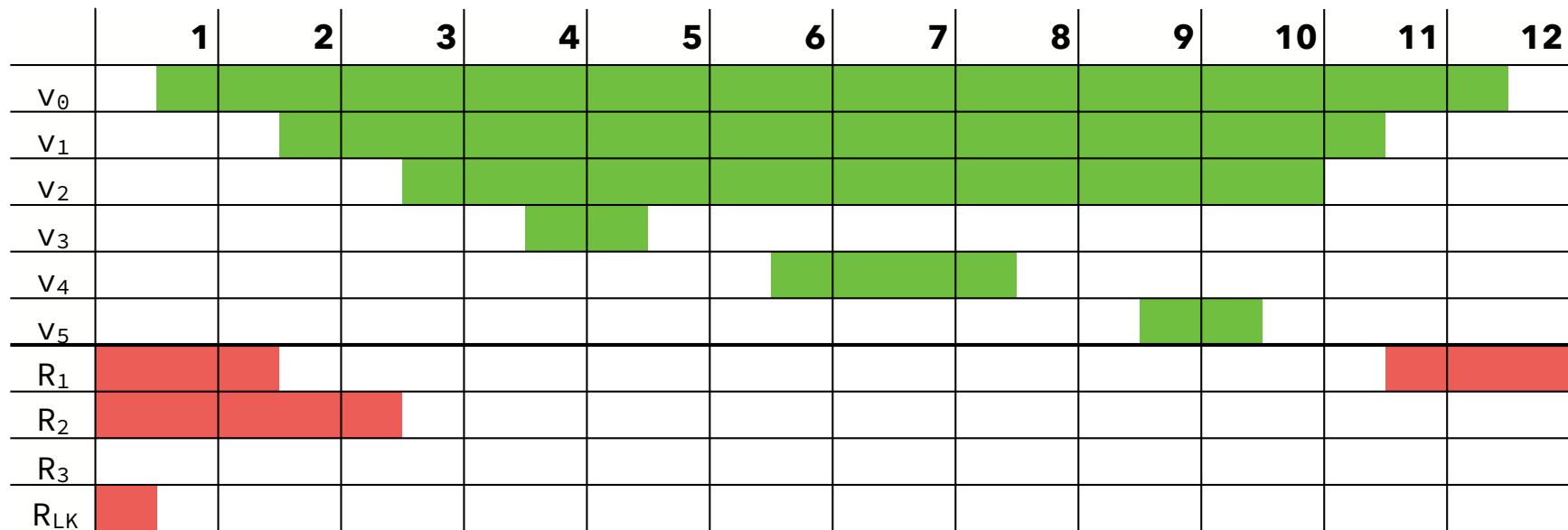
v<sub>5</sub>: [9<sup>+</sup>, 10<sup>-</sup>]

Notation:

$i^+$  entry of instr.  $i$

$i^-$  exit of instr.  $i$

# Linear scan example (4 r.)

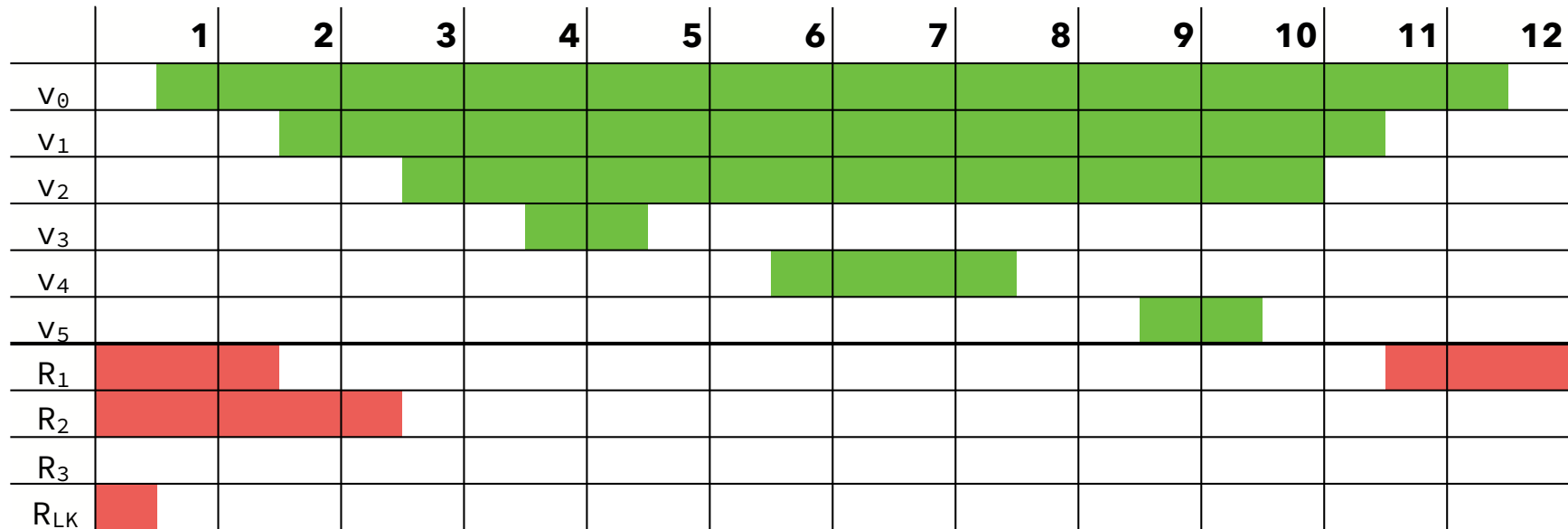


**time active intervals**

**allocation**

---

# Linear scan example (4 r.)



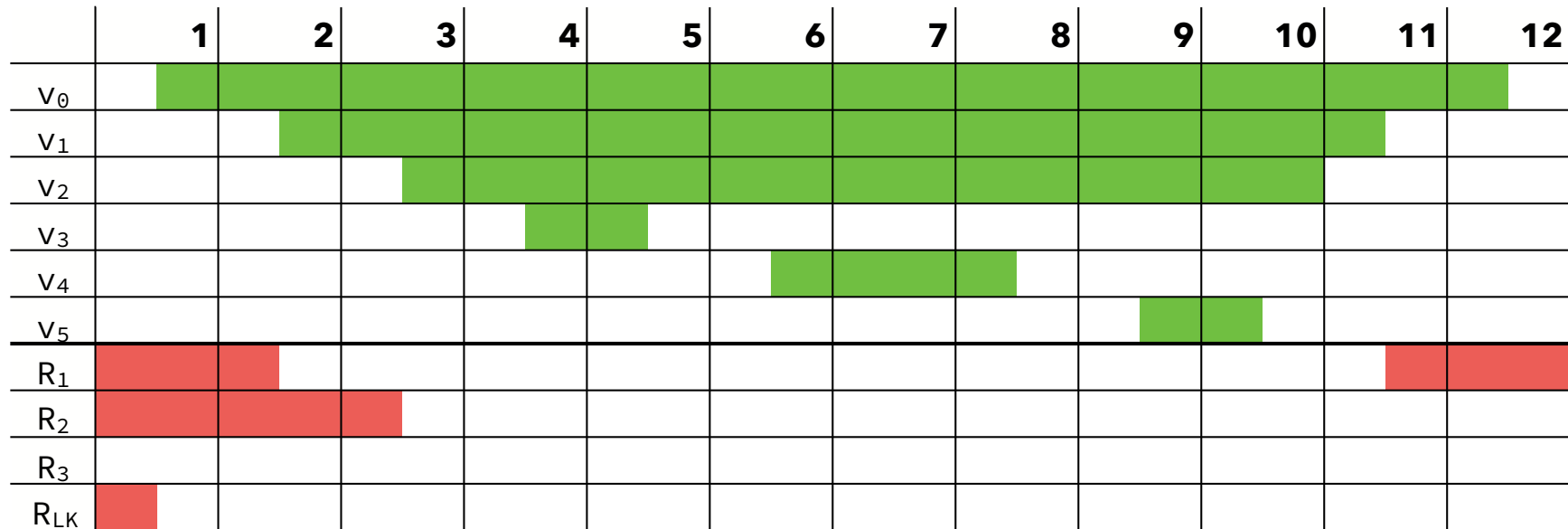
**time active intervals**

**allocation**

1<sup>+</sup> [1<sup>+</sup>,12<sup>-</sup>]

V<sub>0</sub>→R<sub>3</sub>

# Linear scan example (4 r.)



**time active intervals**

**allocation**

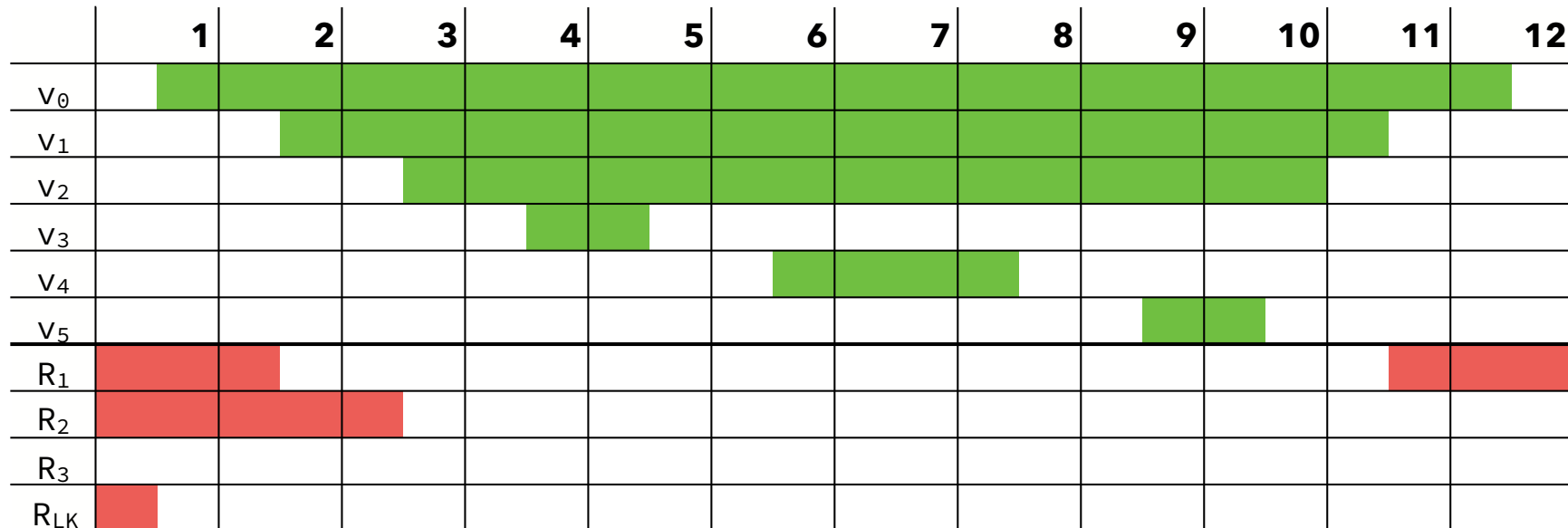
$1^+ [1^+, 12^-]$

$V_0 \rightarrow R_3$

$2^+ [2^+, 11^-], [1^+, 12^-]$

$V_0 \rightarrow R_3, V_1 \rightarrow R_1$

# Linear scan example (4 r.)

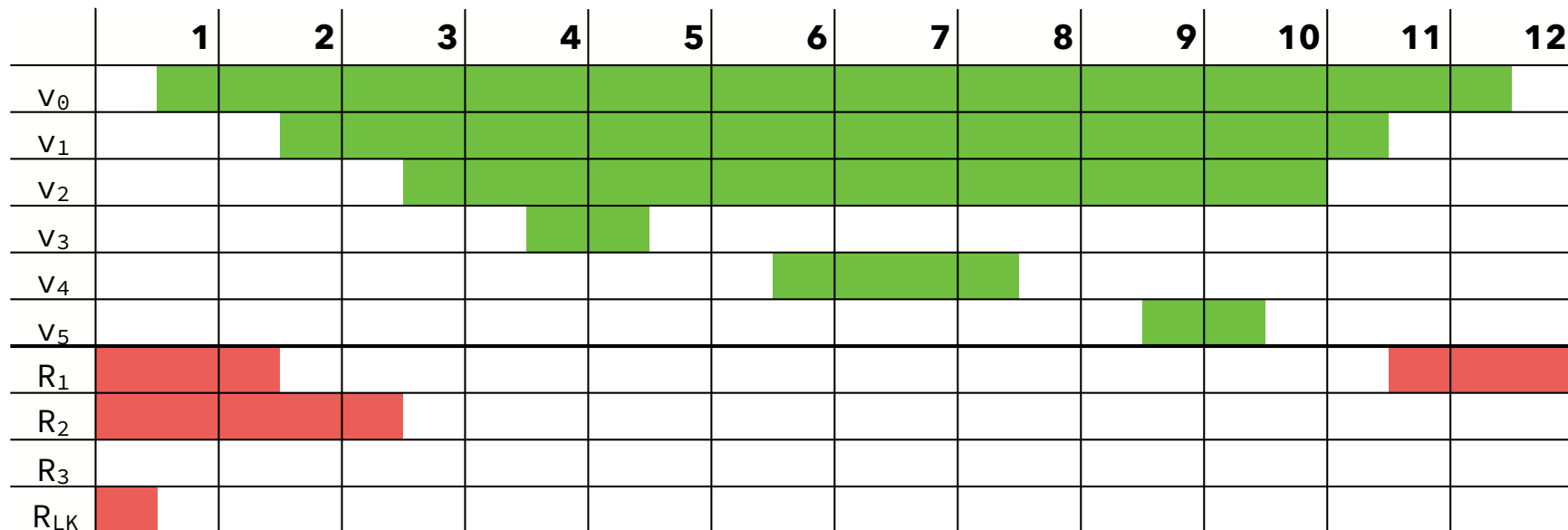


## time active intervals

## allocation

1 <sup>+</sup> [1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub>
2 <sup>+</sup> [2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub>
3 <sup>+</sup> [3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub>

# Linear scan example (4 r.)



## time active intervals

## allocation

1 <sup>+</sup> [1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub>
2 <sup>+</sup> [2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub>
3 <sup>+</sup> [3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub>
4 <sup>+</sup> [4 <sup>+</sup> ,5 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>3</sub> →R <sub>LK</sub>

# Linear scan example (4 r.)

	1	2	3	4	5	6	7	8	9	10	11	12
V <sub>0</sub>	█	█	█	█	█	█	█	█	█	█	█	█
V <sub>1</sub>		█	█	█	█	█	█	█	█	█	█	
V <sub>2</sub>			█	█	█	█	█	█	█	█		
V <sub>3</sub>				█	█							
V <sub>4</sub>						█	█	█				
V <sub>5</sub>									█	█		
R <sub>1</sub>	█	█									█	█
R <sub>2</sub>	█	█	█									
R <sub>3</sub>												
R <sub>LK</sub>	█											

## time active intervals

## allocation

1 <sup>+</sup> [1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub>
2 <sup>+</sup> [2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub>
3 <sup>+</sup> [3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub>
4 <sup>+</sup> [4 <sup>+</sup> ,5 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>3</sub> →R <sub>LK</sub>
6 <sup>+</sup> [6 <sup>+</sup> ,8 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>4</sub> →R <sub>LK</sub>



# Linear scan example (4 r.)

	1	2	3	4	5	6	7	8	9	10	11	12
V <sub>0</sub>	█	█	█	█	█	█	█	█	█	█	█	█
V <sub>1</sub>		█	█	█	█	█	█	█	█	█	█	
V <sub>2</sub>			█	█	█	█	█	█	█	█		
V <sub>3</sub>				█	█							
V <sub>4</sub>						█	█	█				
V <sub>5</sub>									█	█		
R <sub>1</sub>	█	█									█	█
R <sub>2</sub>	█	█	█									
R <sub>3</sub>												
R <sub>LK</sub>	█											

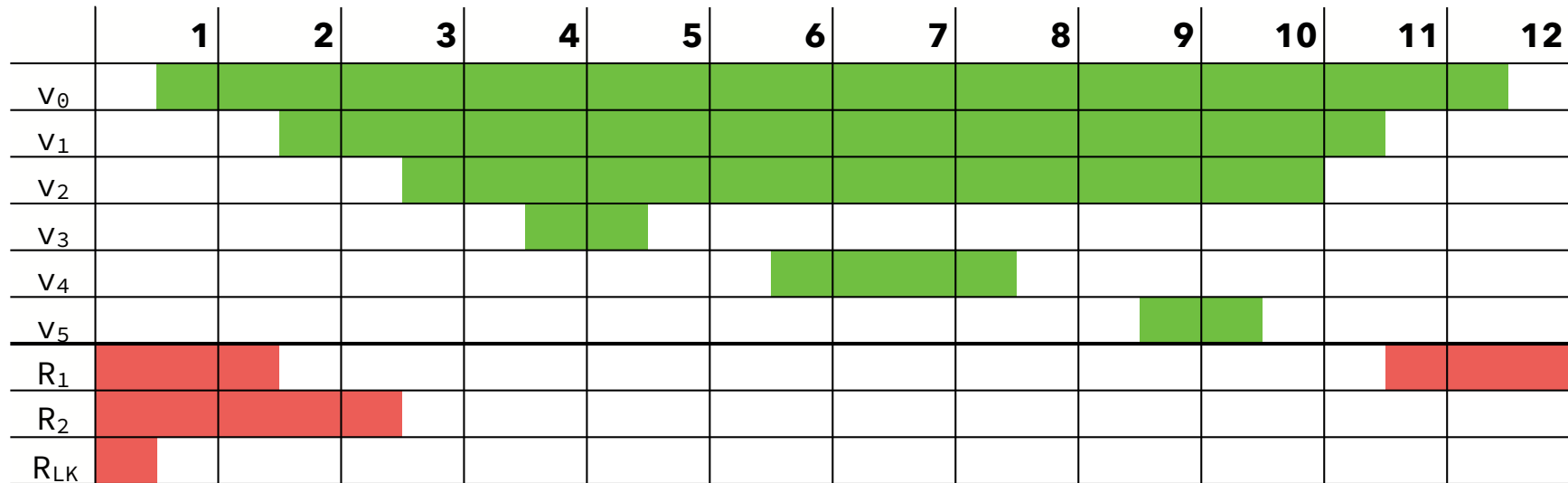
## time active intervals

## allocation

1 <sup>+</sup> [1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub>
2 <sup>+</sup> [2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub>
3 <sup>+</sup> [3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub>
4 <sup>+</sup> [4 <sup>+</sup> ,5 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>3</sub> →R <sub>LK</sub>
6 <sup>+</sup> [6 <sup>+</sup> ,8 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>4</sub> →R <sub>LK</sub>
9 <sup>+</sup> [9 <sup>+</sup> ,10 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>3</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>5</sub> →R <sub>LK</sub>

Result: no spilling

# Linear scan example (3 r.)

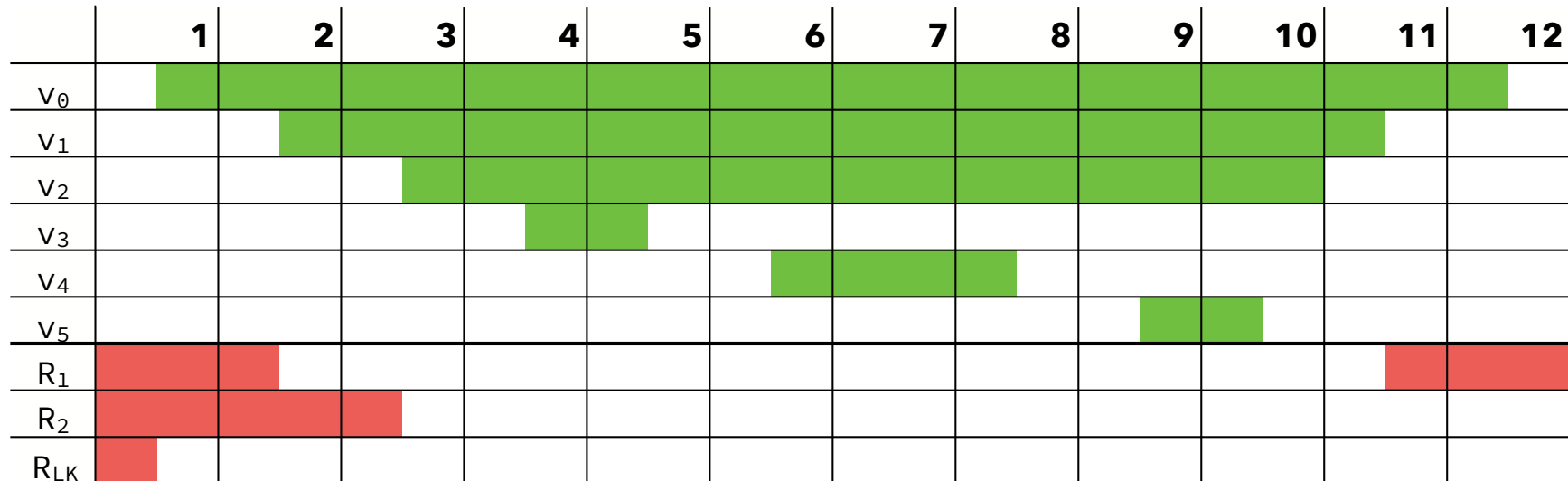


**time active intervals**

**allocation**

---

# Linear scan example (3 r.)



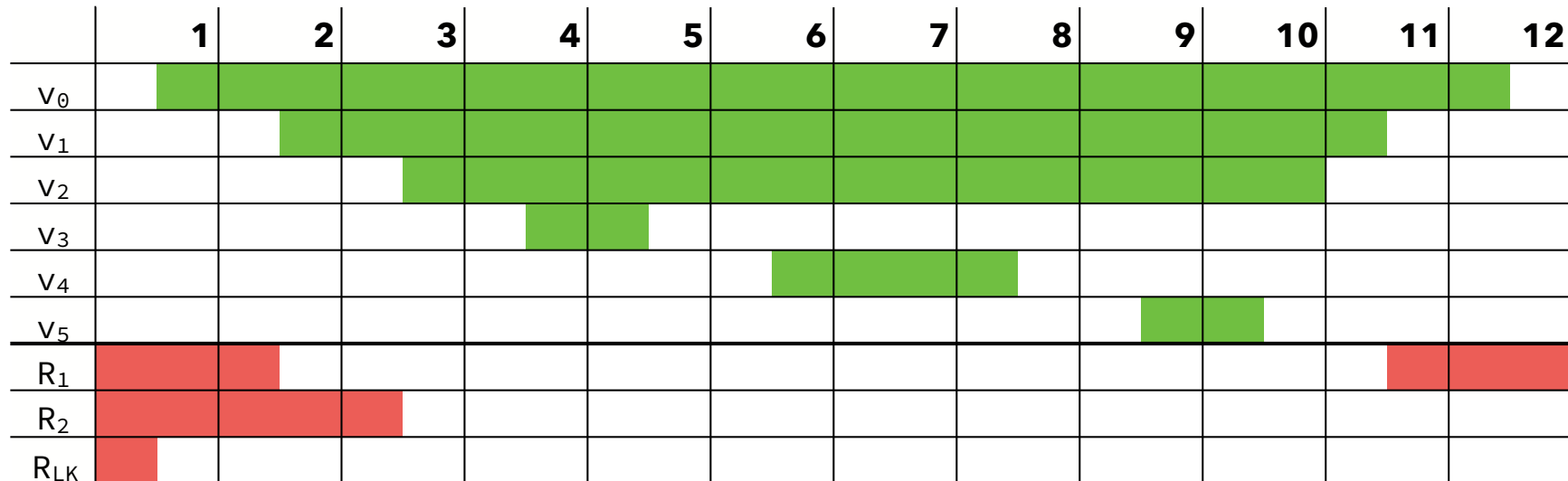
**time active intervals**

**allocation**

1<sup>+</sup> [1<sup>+</sup>,12<sup>-</sup>]

V<sub>0</sub>→R<sub>LK</sub>

# Linear scan example (3 r.)



**time active intervals**

**allocation**

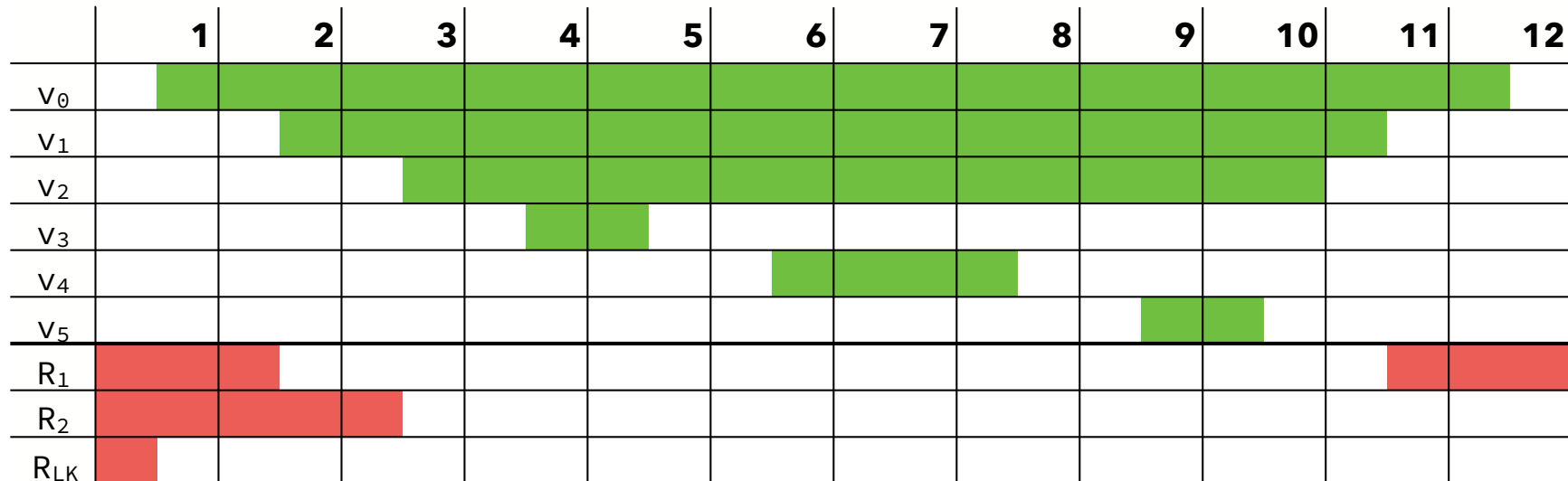
$1^+ [1^+, 12^-]$

$V_0 \rightarrow R_{LK}$

$2^+ [2^+, 11^-], [1^+, 12^-]$

$V_0 \rightarrow R_{LK}, V_1 \rightarrow R_1$

# Linear scan example (3 r.)

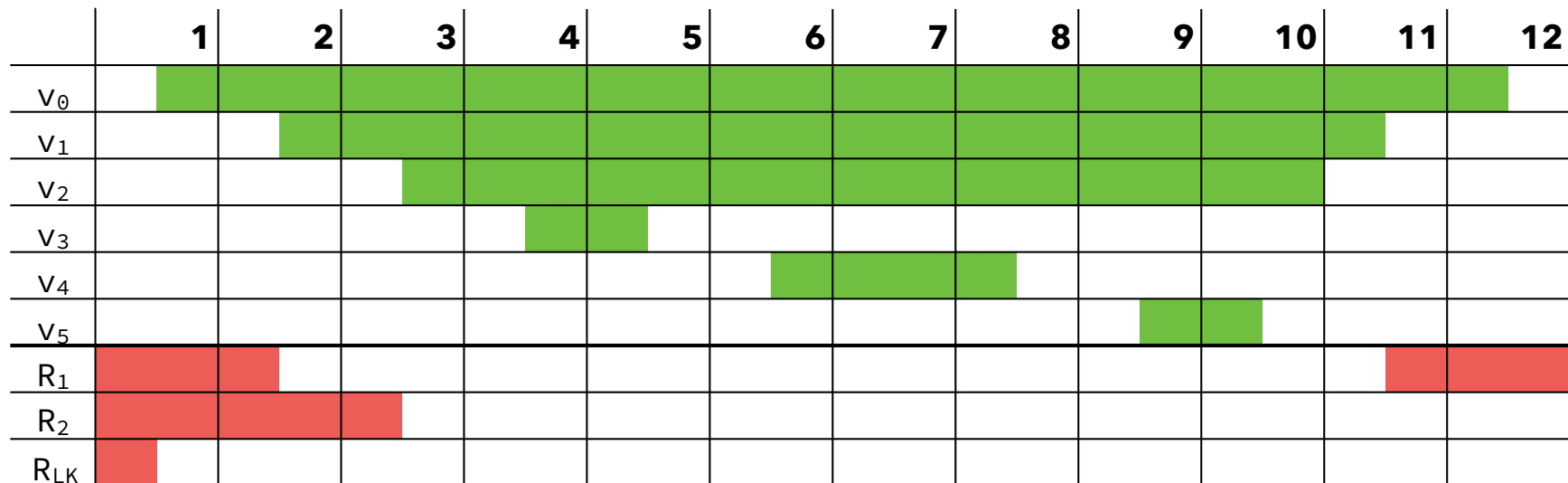


## time active intervals

## allocation

$1^+ [1^+, 12^-]$	$V_0 \rightarrow R_{LK}$
$2^+ [2^+, 11^-], [1^+, 12^-]$	$V_0 \rightarrow R_{LK}, V_1 \rightarrow R_1$
$3^+ [3^+, 10^+], [2^+, 11^-], [1^+, 12^-]$	$V_0 \rightarrow R_{LK}, V_1 \rightarrow R_1, V_2 \rightarrow R_2$

# Linear scan example (3 r.)

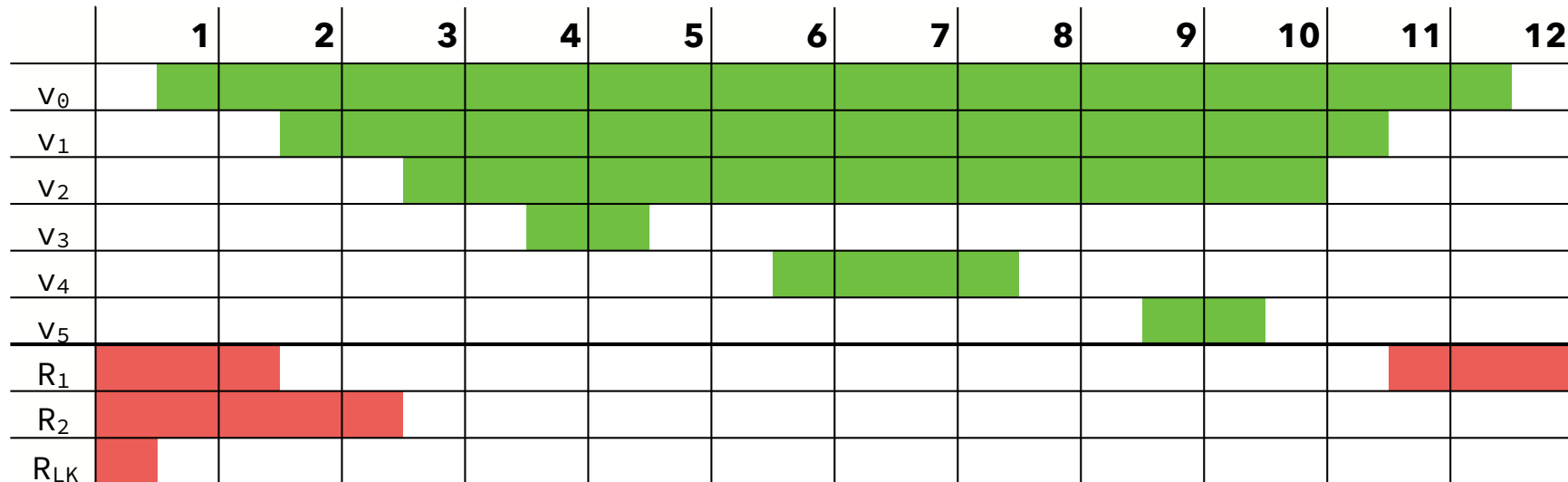


## time active intervals

## allocation

1 <sup>+</sup> [1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>LK</sub>
2 <sup>+</sup> [2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>LK</sub> , V <sub>1</sub> →R <sub>1</sub>
3 <sup>+</sup> [3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>LK</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub>
4 <sup>+</sup> [4 <sup>+</sup> ,5 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ]	V <sub>0</sub> →S, V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>3</sub> →R <sub>LK</sub>

# Linear scan example (3 r.)



## time active intervals

## allocation

1 <sup>+</sup> [1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>LK</sub>
2 <sup>+</sup> [2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>LK</sub> , V <sub>1</sub> →R <sub>1</sub>
3 <sup>+</sup> [3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ],[1 <sup>+</sup> ,12 <sup>-</sup> ]	V <sub>0</sub> →R <sub>LK</sub> , V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub>
4 <sup>+</sup> [4 <sup>+</sup> ,5 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ]	V <sub>0</sub> →S, V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>3</sub> →R <sub>LK</sub>
6 <sup>+</sup> [6 <sup>+</sup> ,8 <sup>-</sup> ],[3 <sup>+</sup> ,10 <sup>+</sup> ],[2 <sup>+</sup> ,11 <sup>-</sup> ]	V <sub>0</sub> →S, V <sub>1</sub> →R <sub>1</sub> , V <sub>2</sub> →R <sub>2</sub> , V <sub>4</sub> →R <sub>LK</sub>

# Linear scan example (3 r.)

	1	2	3	4	5	6	7	8	9	10	11	12
$V_0$	█	█	█	█	█	█	█	█	█	█	█	█
$V_1$		█	█	█	█	█	█	█	█	█	█	
$V_2$			█	█	█	█	█	█	█	█		
$V_3$				█	█							
$V_4$						█	█	█				
$V_5$									█	█		
$R_1$	█	█									█	█
$R_2$	█	█	█									
$R_{LK}$	█											

## time active intervals

## allocation

$1^+ [1^+, 12^-]$	$V_0 \rightarrow R_{LK}$
$2^+ [2^+, 11^-], [1^+, 12^-]$	$V_0 \rightarrow R_{LK}, V_1 \rightarrow R_1$
$3^+ [3^+, 10^+], [2^+, 11^-], [1^+, 12^-]$	$V_0 \rightarrow R_{LK}, V_1 \rightarrow R_1, V_2 \rightarrow R_2$
$4^+ [4^+, 5^-], [3^+, 10^+], [2^+, 11^-]$	$V_0 \rightarrow S, V_1 \rightarrow R_1, V_2 \rightarrow R_2, V_3 \rightarrow R_{LK}$
$6^+ [6^+, 8^-], [3^+, 10^+], [2^+, 11^-]$	$V_0 \rightarrow S, V_1 \rightarrow R_1, V_2 \rightarrow R_2, V_4 \rightarrow R_{LK}$
$9^+ [9^+, 10^-], [3^+, 10^+], [2^+, 11^-]$	$V_0 \rightarrow S, V_1 \rightarrow R_1, V_2 \rightarrow R_2, V_5 \rightarrow R_{LK}$

Result:  $v_0$  is spilled *during its whole life time!*



# Linear scan improvements

The basic linear scan algorithm is very simple but still produces reasonably good code. It can be – and has been – improved in many ways:

- the liveness information about virtual registers can be described using a sequence of disjoint intervals instead of a single one,
- virtual registers can be spilled for only a part of their whole life time,
- more sophisticated heuristics can be used to select the virtual register to spill,
- etc.