

Example: Square roots with Newton's method

Task

We will define in this session a function

```
/** Calculates the square root of parameter x */  
def sqrt(x: Double): Double = ...
```

The classical way to achieve this is by successive approximations using Newton's method.

Method

To compute $\text{sqrt}(x)$:

- ▶ Start with an initial *estimate* y (let's pick $y = 1$).
- ▶ Repeatedly improve the estimate by taking the mean of y and x/y .

Example:

Estimation	Quotient	Mean
1	$2 / 1 = 2$	1.5
1.5	$2 / 1.5 = 1.333$	1.4167
1.4167	$2 / 1.4167 = 1.4118$	1.4142
1.4142

Implementation in Scala (1)

First, define a function which computes one iteration step

```
def sqrtIter(guess: Double, x: Double): Double =  
  if (isGoodEnough(guess, x)) guess  
  else sqrtIter(improve(guess, x), x)
```

Note that `sqrtIter` is *recursive*, its right-hand side calls itself.

Recursive functions need an explicit return type in Scala.

For non-recursive functions, the return type is optional

Implementation in Scala (2)

Second, define a function `improve` to improve an estimate and a test to check for termination:

```
def improve(guess: Double, x: Double) =  
  (guess + x / guess) / 2  
  
def isGoodEnough(guess: Double, x: Double) =  
  abs(guess * guess - x) < 0.001
```

Implementation in Scala (3)

Third, define the sqrt function:

```
def sqrt(x: Double) = sqrtIter(1.0, x)
```

Exercise

1. The `isGoodEnough` test is not very precise for small numbers and can lead to non-termination for very large numbers. Explain why.
2. Design a different version of `isGoodEnough` that does not have these problems.
3. Test your version with some very very small and large numbers, e.g.

`0.001`

`0.1e-20`

`1.0e20`

`1.0e50`