



ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Data-Parallel Operations

Parallel Programming and Data Analysis

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Scala Collection Recap

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Easy way to manipulate data – a large number of combinator methods.

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(1 until 1000).filter(n => n % 3 == 0).foldLeft(0)(_ + _)
```

Question:

What does this expression evaluate to?

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(1 until 1000).filter(n => n % 3 == 0).foldLeft(0)(_ + _)
```

Question:

What does this expression evaluate to?

- ▶ numbers smaller than 1000 that are divisible by 3
- ▶ a sum of numbers smaller than 1000 that are divisible by 3
- ▶ a sum of numbers smaller than 1000 modulo 3

Scala Collection Recap

```
(100 to 999).flatMap(i => (i to 999).map(i * _))  
  .filter(n => n.toString == n.toString.reverse).max
```

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Question:

What does this expression evaluate to?

- ▶ the largest 3-digit palindrome
- ▶ the largest product of 3-digit palindromes
- ▶ the largest palindrome that is a product of 3-digit numbers

Data-Parallel Collections

The `par` method transforms a sequential Scala collection into a parallel one.

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(100 to 999).par.flatMap(i => (i to 999).map(i * _))  
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- ▶ the same operations as sequential collections, but most execute in parallel
- ▶ *deterministic* as long as they are used functionally

Non-Parallelizable Operations

Task: implement the method `sum` using the `foldLeft` method.

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Does this implementation execute in parallel?

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Does this implementation execute in parallel?

Why not?

Non-Parallelizable Operations

Let's examine the `foldLeft` signature:

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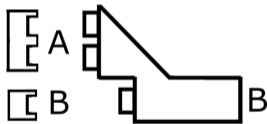
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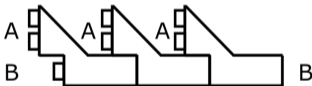
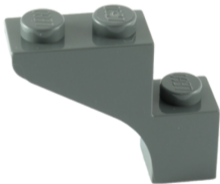
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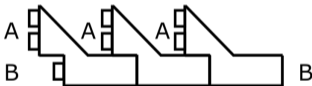
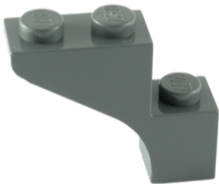
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Operations foldRight, reduceLeft, reduceRight, scanLeft and scanRight similarly must process elements sequentially.

The fold Operation

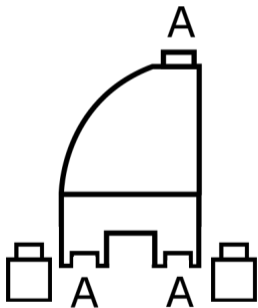
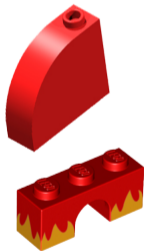
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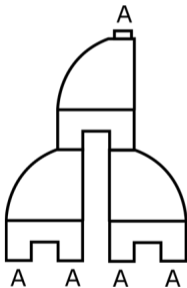
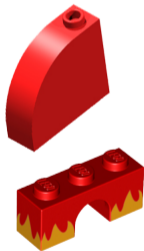
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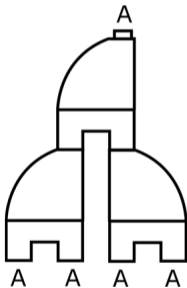
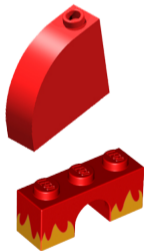
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The fold operation can process the elements in a reduction tree, so it can execute in parallel.

Use-cases of the fold Operation

Implement the sum method:

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Use-cases of the fold Operation

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def sum(xs: Array[Int]): Int = {  
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```

Implement the max method:

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def max(xs: Array[Int]): Int = {  
  xs.par.fold(Int.MinValue)(math.max)  
}
```

Preconditions of the fold Operation

Given a list of "paper", "rock" and "scissors" strings, find out who won:

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Array("paper", "rock", "paper", "scissors")
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Given a list of "paper", "rock" and "scissors" strings, find out who won:

```
Array("paper", "rock", "paper", "scissors")  
  .par.fold("")(play)
```

```
def play(a: String, b: String): String = List(a, b).sorted match {  
  case List("paper", "scissors") => "scissors"  
  case List("paper", "rock")     => "paper"  
  case List("rock", "scissors")  => "rock"  
  case List(a, b) if a == b      => a  
  case List("", b)               => b  
}
```

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play("paper", play("rock", play("paper", "scissors"))) == "paper"
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Why does this happen?

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Why does this happen?

The play operator is *commutative*, but not *associative*.

Preconditions of the fold Operation

In order for the fold operation to work correctly, the following relations must hold:

$$f(a, f(b, c)) == f(f(a, b), c)$$

$$f(z, a) == f(a, z) == a$$

We say that the neutral element z and the binary operator f must form a *monoid*.

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We say that the neutral element z and the binary operator f must form a *monoid*.

Commutativity does not matter for fold – the following relation is not necessary:

$$f(a, b) == f(b, a)$$

Limitations of the fold Operation

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```
Array('E', 'P', 'F', 'L').par  
  .fold(0)((count, c) => if (isVowel(c)) count + 1 else count)
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Limitations of the fold Operation

Given an array of characters, use fold to return the vowel count:

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The fold operation can only produce values of the same type as the collection that it is called on.

The foldLeft operation is *more expressive* than fold. Sanity check:

```
def fold(z: A)(op: (A, A) => A): A = foldLeft[A](z)(op)
```

Implementing foldLeft using fold is not so straightforward.

The aggregate Operation

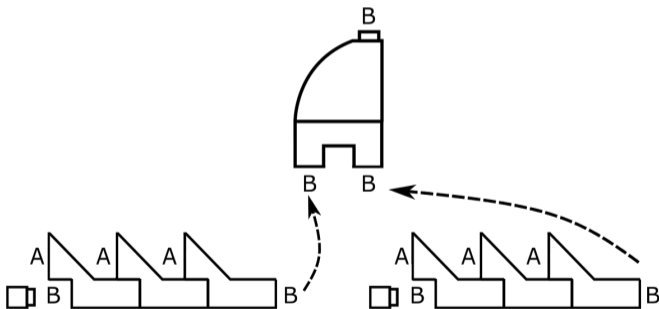
Let's examine the aggregate signature:

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A combination of foldLeft and fold.

Using the aggregate Operation

Count the number of vowels in a character array:

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```
Array('E', 'P', 'F', 'L').aggregate(0)(  
  (count, c) => if (isVowel(c)) count + 1 else count,  
  _ + _  
)
```

Parallelizable Operations

Alternatively, consider the `mapFold` method. Can you implement `mapFold` using only `foldLeft`?

```
def mapFold[B](f: A => B)(z: B)(g: (B, B) => B): B
```

Parallelizable Operations

Consider the `mapFold` method. Can you implement `mapFold` using only `foldLeft`?

```
def mapFold[B](f: A => B)(z: B)(g: (B, B) => B): B =  
  foldLeft(z)((b, a) => g(b, f(a)))
```

This particular implementation of `mapFold`, however, does not run in parallel.

Parallelizable Operations

For those who like challenges, implement:

```
def foldLeft[B](z: B)(op: (B, A) => B): B
```

using only the `mapFold` operation.

Can this method execute in parallel?

The Transformer Operations

So far, we saw the *accessor* combinators.

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They should be familiar from sequential programming.