

## Currying

## Motivation

Look again at the summation functions:

```
def sumInts(a: Int, b: Int)      = sum(x => x, a, b)
def sumCubes(a: Int, b: Int)    = sum(x => x * x * x, a, b)
def sumFactorials(a: Int, b: Int) = sum(fact, a, b)
```

### Question

Note that `a` and `b` get passed unchanged from `sumInts` and `sumCubes` into `sum`.

Can we be even shorter by getting rid of these parameters?

# Functions Returning Functions

Let's rewrite sum as follows.

```
def sum(f: Int => Int): (Int, Int) => Int = {  
  def sumF(a: Int, b: Int): Int =  
    if (a > b) 0  
    else f(a) + sumF(a + 1, b)  
  sumF  
}
```

sum is now a function that returns another function.

The returned function sumF applies the given function parameter f and sums the results.

## Stepwise Applications

We can then define:

```
def sumInts      = sum(x => x)
def sumCubes     = sum(x => x * x * x)
def sumFactorials = sum(factorial)
```

These functions can in turn be applied like any other function:

```
sumCubes(1, 10) + sumFactorials(10, 20)
```

## Consecutive Stepwise Applications

In the previous example, can we avoid the `sumInts`, `sumCubes`, ... middlemen?

Of course:

```
sum (cube) (1, 10)
```

## Consecutive Stepwise Applications

In the previous example, can we avoid the `sumInts`, `sumCubes`, ... middlemen?

Of course:

```
sum (cube) (1, 10)
```

- ▶ `sum(cube)` applies `sum` to `cube` and returns the *sum of cubes* function.
- ▶ `sum(cube)` is therefore equivalent to `sumCubes`.
- ▶ This function is next applied to the arguments `(1, 10)`.

## Consecutive Stepwise Applications

In the previous example, can we avoid the `sumInts`, `sumCubes`, ... middlemen?

Of course:

```
sum (cube) (1, 10)
```

- ▶ `sum(cube)` applies `sum` to `cube` and returns the *sum of cubes* function.
- ▶ `sum(cube)` is therefore equivalent to `sumCubes`.
- ▶ This function is next applied to the arguments `(1, 10)`.

Generally, function application associates to the left:

```
sum(cube)(1, 10) == (sum (cube)) (1, 10)
```

## Multiple Parameter Lists

The definition of functions that return functions is so useful in functional programming that there is a special syntax for it in Scala.

For example, the following definition of `sum` is equivalent to the one with the nested `sumF` function, but shorter:

```
def sum(f: Int => Int)(a: Int, b: Int): Int =  
  if (a > b) 0 else f(a) + sum(f)(a + 1, b)
```



## Expansion of Multiple Parameter Lists

In general, a definition of a function with multiple parameter lists

$$\text{def } f(\text{args}_1)\dots(\text{args}_n) = E$$

where  $n > 1$ , is equivalent to

$$\text{def } f(\text{args}_1)\dots(\text{args}_{n-1}) = \{\text{def } g(\text{args}_n) = E; g\}$$

where  $g$  is a fresh identifier. Or for short:

$$\text{def } f(\text{args}_1)\dots(\text{args}_{n-1}) = (\text{args}_n \Rightarrow E)$$

## Expansion of Multiple Parameter Lists (2)

By repeating the process  $n$  times

$$\text{def } f(\text{args}_1)\dots(\text{args}_{n-1})(\text{args}_n) = E$$

is shown to be equivalent to

$$\text{def } f = (\text{args}_1 \Rightarrow (\text{args}_2 \Rightarrow \dots(\text{args}_n \Rightarrow E)\dots))$$

This style of definition and function application is called *currying*, named for its instigator, Haskell Brooks Curry (1900-1982), a twentieth century logician.

In fact, the idea goes back even further to Schönfinkel and Frege, but the term “currying” has stuck.

## More Function Types

Question: Given,

```
def sum(f: Int => Int)(a: Int, b: Int): Int = ...
```

What is the type of sum ?

## More Function Types

Question: Given,

```
def sum(f: Int => Int)(a: Int, b: Int): Int = ...
```

What is the type of sum ?

**Answer:**

```
(Int => Int) => (Int, Int) => Int
```

Note that functional types associate to the right. That is to say that

```
Int => Int => Int
```

is equivalent to

```
Int => (Int => Int)
```

## Exercise

1. Write a product function that calculates the product of the values of a function for the points on a given interval.
2. Write factorial in terms of product.
3. Can you write a more general function, which generalizes both sum and product?