

## Higher-order List Functions

## Recurring Patterns for Computations on Lists

The examples have shown that functions on lists often have similar structures.

We can identify several recurring patterns, like,

- ▶ transforming each element in a list in a certain way,
- ▶ retrieving a list of all elements satisfying a criterion,
- ▶ combining the elements of a list using an operator.

Functional languages allow programmers to write generic functions that implement patterns such as these using **higher-order functions**.

## Applying a Function to Elements of a List

A common operation is to transform each element of a list and then return the list of results.

For example, to multiply each element of a list by the same factor, you could write:

```
def scaleList(xs: List[Double], factor: Double): List[Double] = xs match {  
  case Nil      => xs  
  case y :: ys => y * factor :: scaleList(ys, factor)  
}
```

# Map

This scheme can be generalized to the method `map` of the `List` class. A simple way to define `map` is as follows:

```
abstract class List[T] { ...
  def map[U](f: T => U): List[U] = this match {
    case Nil      => this
    case x :: xs => f(x) :: xs.map(f)
  }
```

(in fact, the actual definition of `map` is a bit more complicated, because it is tail-recursive, and also because it works for arbitrary collections, not just lists).

Using `map`, `scaleList` can be written more concisely.

```
def scaleList(xs: List[Double], factor: Double) =
  xs map (x => x * factor)
```

## Exercise

Consider a function to square each element of a list, and return the result. Complete the two following equivalent definitions of `squareList`.

```
def squareList(xs: List[Int]): List[Int] = xs match {  
  case Nil      => ???  
  case y :: ys => ???  
}
```

```
def squareList(xs: List[Int]): List[Int] =  
  xs map ???
```

## Exercise

Consider a function to square each element of a list, and return the result. Complete the two following equivalent definitions of `squareList`.

```
def squareList(xs: List[Int]): List[Int] = xs match {  
  case Nil      =>  
  case y :: ys =>  
}
```

```
def squareList(xs: List[Int]): List[Int] =  
  xs map
```

## Filtering

Another common operation on lists is the selection of all elements satisfying a given condition. For example:

```
def posElems(xs: List[Int]): List[Int] = xs match {  
  case Nil      => xs  
  case y :: ys => if (y > 0) y :: posElems(ys) else posElems(ys)  
}
```

## Filter

This pattern is generalized by the method `filter` of the `List` class:

```
abstract class List[T] {  
  ...  
  def filter(p: T => Boolean): List[T] = this match {  
    case Nil => this  
    case x :: xs => if (p(x)) x :: xs.filter(p) else xs.filter(p)  
  }  
}
```

Using `filter`, `posElems` can be written more concisely.

```
def posElems(xs: List[Int]): List[Int] =  
  xs filter (x => x > 0)
```

## Variations of Filter

Besides filter, there are also the following methods that extract sublists based on a predicate:

- `xs filterNot p` Same as `xs filter (x => !p(x))`; The list consisting of those elements of `xs` that do not satisfy the predicate `p`.
- `xs partition p` Same as `(xs filter p, xs filterNot p)`, but computed in a single traversal of the list `xs`.
- `xs takeWhile p` The longest prefix of list `xs` consisting of elements that all satisfy the predicate `p`.
- `xs dropWhile p` The remainder of the list `xs` after any leading elements satisfying `p` have been removed.
- `xs span p` Same as `(xs takeWhile p, xs dropWhile p)` but computed in a single traversal of the list `xs`.

## Exercise

Write a function `pack` that packs consecutive duplicates of list elements into sublists. For instance,

```
pack(List("a", "a", "a", "b", "c", "c", "a"))
```

should give

```
List(List("a", "a", "a"), List("b"), List("c", "c"), List("a")).
```

You can use the following template:

```
def pack[T](xs: List[T]): List[List[T]] = xs match {  
  case Nil      => Nil  
  case x :: xs1 => ???  
}
```

## Exercise

Using `pack`, write a function `encode` that produces the run-length encoding of a list.

The idea is to encode  $n$  consecutive duplicates of an element  $x$  as a pair  $(x, n)$ . For instance,

```
encode(List("a", "a", "a", "b", "c", "c", "a"))
```

should give

```
List(("a", 3), ("b", 1), ("c", 2), ("a", 1)).
```