Combinatorial Search and For-Expressions

Handling Nested Sequences

We can extend the usage of higher order functions on sequences to many calculations which are usually expressed using nested loops.

Example: Given a positive integer `n`, find all pairs of positive integers `i` and `j`, with `1 <= j < i < n` such that `i + j` is prime.

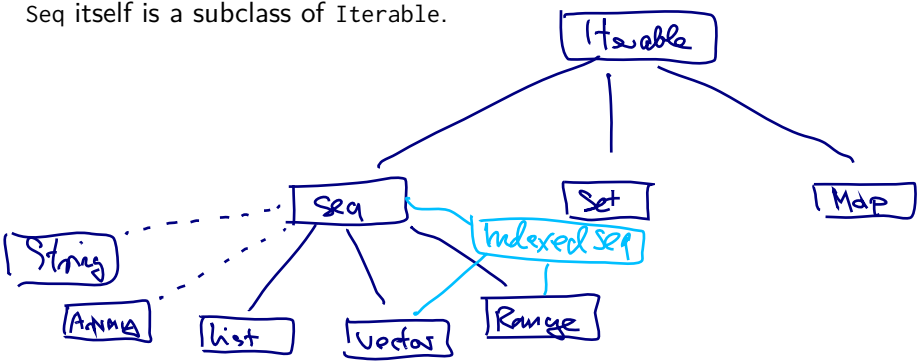
For example, if `n = 7`, the sought pairs are

i	2	3	4	4	5	6	6
j	1	2	1	3	2	1	5
$i+j$	3	5	5	7	7	7	11

Collection Hierarchy

A common base class of `List` and `Vector` is `Seq`, the class of all *sequences*.

`Seq` itself is a subclass of `Iterable`.



Algorithm

A natural way to do this is to:

- Generate the sequence of all pairs of integers `(i, j)` such that `1 <= j < i < n`.
- Filter the pairs for which `i + j` is prime.

One natural way to generate the sequence of pairs is to:

- Generate all the integers `i` between `1` and `n` (excluded).
- For each integer `i`, generate the list of pairs `(i, 1)`, `...`, `(i, i-1)`.

This can be achieved by combining `until` and `map`:

```
(1 until n) map (i =>
  (1 until i) map (j => (i, j)))
```

Generate Pairs

The previous step gave a sequence of sequences, let's call it `xss`.

We can combine all the sub-sequences using `foldRight` with `++`:

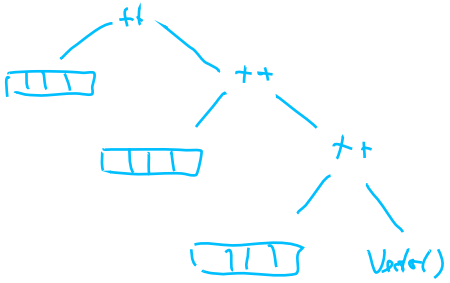
```
(xss foldRight Seq[Int]())(_ ++ _)
```

Or, equivalently, we use the built-in method `flatten`

```
xss.flatten
```

This gives:

```
((1 until n) map (i =>
  (1 until i) map (j => (i, j)))).flatten
```



Generate Pairs (2)

Here's a useful law:

```
xs flatMap f  =  (xs map f).flatten
```

Hence, the above expression can be simplified to

```
(1 until n) flatMap (i =>
    (1 until i) map (j => (i, j)))
```

Assembling the pieces

By reassembling the pieces, we obtain the following expression:

```
(1 until n) flatMap (i =>
   (1 until i) map (j => (i, j))) filter ( pair =>
      isPrime(pair._1 + pair._2))
```

This works, but makes most people's head hurt.

Is there a simpler way?

For-Expressions

Higher-order functions such as `map`, `flatMap` or `filter` provide powerful constructs for manipulating lists.

But sometimes the level of abstraction required by these function make the program difficult to understand.

In this case, Scala's `for` expression notation can help.

For-Expression Example

Let `persons` be a list of elements of class `Person`, with fields `name` and `age`.

```
case class Person(name: String, age: Int)
```

To obtain the names of persons over 20 years old, you can write:

```
for ( p <- persons if p.age > 20 ) yield p.name
```

which is equivalent to:

```
persons filter (p => p.age > 20) map (p => p.name)
```

The for-expression is similar to loops in imperative languages, except that it builds a list of the results of all iterations.

Syntax of For

A for-expression is of the form

```
for ( s ) yield e
```

where `s` is a sequence of *generators* and *filters*, and `e` is an expression whose value is returned by an iteration.

- A *generator* is of the form `p <- e`, where `p` is a pattern and `e` an expression whose value is a collection.
- A *filter* is of the form `if f` where `f` is a boolean expression.
- The sequence must start with a generator.
- If there are several generators in the sequence, the last generators vary faster than the first.

Instead of `( s )`, braces `{ s }` can also be used, and then the sequence of generators and filters can be written on multiple lines without requiring semicolons.

Use of For

Here are two examples which were previously solved with higher-order functions:

Given a positive integer `n`, find all the pairs of positive integers `(i, j)` such that `1 <= j < i < n`, and `i + j` is prime.

```
for {
  i <- 1 until n
  j <- 1 until i
  if isPrime(i + j)
} yield (i, j)
```

Exercise

Write a version of `scalarProduct` (see last session) that makes use of a `for`:

```
def scalarProduct(xs: List[Double], ys: List[Double]) : Double =
```

(for ((x,y) ← xs zip ys) yield x*y).sum

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