

Combinatorial Search Example

Sets

Sets are another basic abstraction in the Scala collections.

A set is written analogously to a sequence:

```
val fruit = Set("apple", "banana", "pear")  
val s = (1 to 6).toSet
```

Most operations on sequences are also available on sets:

```
s map (_ + 2)  
fruit filter (_.startsWith == "app")  
s.nonEmpty
```

(see Iterables Scaladoc for a list of all supported operations)

Sets vs Sequences

The principal differences between sets and sequences are:

1. Sets are unordered; the elements of a set do not have a predefined order in which they appear in the set
2. sets do not have duplicate elements:

```
s map (_ / 2)      // Set(2, 0, 3, 1)
```

3. The fundamental operation on sets is contains:

```
s contains 5      // true
```

Example: N-Queens

The eight queens problem is to place eight queens on a chessboard so that no queen is threatened by another.

- ▶ In other words, there can't be two queens in the same row, column, or diagonal.

We now develop a solution for a chessboard of any size, not just 8.

One way to solve the problem is to place a queen on each row.

Once we have placed $k - 1$ queens, one must place the k th queen in a column where it's not "in check" with any other queen on the board.

Algorithm

We can solve this problem with a recursive algorithm:

- ▶ Suppose that we have already generated all the solutions consisting of placing $k-1$ queens on a board of size n .
- ▶ Each solution is represented by a list (of length $k-1$) containing the numbers of columns (between 0 and $n-1$).
- ▶ The column number of the queen in the $k-1$ th row comes first in the list, followed by the column number of the queen in row $k-2$, etc.
- ▶ The solution set is thus represented as a set of lists, with one element for each solution.
- ▶ Now, to place the k th queen, we generate all possible extensions of each solution preceded by a new queen:

Implementation

```
def queens(n: Int) = {  
  def placeQueens(k: Int): Set[List[Int]] = {  
    if (k == 0) Set(List())  
    else  
      for {  
        queens <- placeQueens(k - 1)  
        col <- 0 until n  
        if isSafe(col, queens)  
      } yield col :: queens  
  }  
  placeQueens(n)  
}
```

Exercise

Write a function

```
def isSafe(col: Int, queens: List[Int]): Boolean
```

which tests if a queen placed in an indicated column `col` is secure amongst the other placed queens.

It is assumed that the new queen is placed in the next available row after the other placed queens (in other words: in row `queens.length`).

$\text{list}(0, 3, 1) \rightarrow \text{list}((2, 0), (1, 3), (0, 1))$